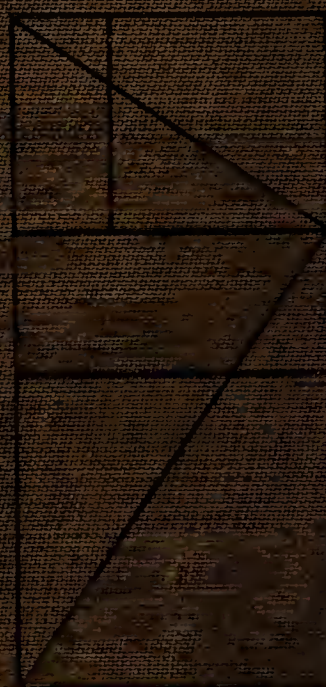


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THE PARTHENON
AND OTHER GREEK TEMPLES
THEIR DYNAMIC SYMMETRY
BY JAY HAMBIDGE



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ADDISON GALLERY
COLLECTION

THE PARTHENON

PUBLISHED UNDER THE AUSPICES OF
THE SCHOOL OF THE FINE ARTS, YALE UNIVERSITY
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IN MEMORY OF
RUTHERFORD TROWBRIDGE

THE PARTHENON AND OTHER GREEK TEMPLES THEIR DYNAMIC SYMMETRY

BY JAY HAMBIDGE

AUTHOR OF "DYNAMIC SYMMETRY: THE GREEK VASE,"
"DYNAMIC SYMMETRY IN COMPOSITION AS USED BY
THE ARTISTS," AND OF "THE DIAGONAL"

WITH A PREFACE BY L. D. CASKEY

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THE present volume is the second work published on the Rutherford Trowbridge Memorial Publication Fund. This Foundation was established in May, 1920, through a gift to Yale University by his widow in memory of Rutherford Trowbridge, Esq., of New Haven, who died December 18, 1918, and who in 1899 had established the Thomas Rutherford Trowbridge Memorial Lectureship Fund in the School of the Fine Arts at Yale. It was in a series of lectures delivered on this Foundation that the material comprised in this volume was first given to the public. By the establishment of the Rutherford Trowbridge Memorial Publication Fund, the University has been enabled to make available for a much wider audience the work growing out of lectures given at Yale through the generosity of one who sought always to render service to the community in which he lived; and, through its university, to the world.

CONTENTS

LIST OF PHOTOGRAPHIC ILLUSTRATIONS xi

PREFACE xiii

INTRODUCTION I

CHAPTER ONE: "THE GREEK TEMPLE" 7

CHAPTER TWO: THE FAÇADE AND ITS DIVISIONS 14

CHAPTER THREE: THE ENTABLATURE 20

CHAPTER FOUR: THE PARTHENON COLUMN 27

CHAPTER FIVE: THE TEMPLE FLOOR 37

CHAPTER SIX: THE TEMPLE FLOOR AND ELEVATION 43

CHAPTER SEVEN: THE PANATHENAIC FRIEZE AND THE CELLA . . . 48

CHAPTER EIGHT: THE NAOS 51

CHAPTER NINE: THE ANTEFIX 56

CHAPTER TEN: THE GREEK SPIRAL 66

CHAPTER ELEVEN: ANOTHER TEMPLE BY THE ARCHITECT OF THE
PARTHENON 78

CHAPTER TWELVE: THE DORIC TEMPLES AT AEGINA AND OLYMPIA 83

APPENDIX: NOTES I-VIII 88

PROOF BY ARITHMETIC OF THE DIAGRAMS IN THE
TEXT 96

LIST OF PHOTOGRAPHIC ILLUSTRATIONS

	<i>Opposite page</i>
FIGURE 33. UNFINISHED CAPITAL OF THE EARLIER PARTHE- NON	28
FIGURE 34. A VERY EARLY UNFINISHED DORIC CAPITAL . . .	28
FIGURE 55. A SLAB FROM THE FRIEZE OF THE PARTHENON . .	48
FIGURE 65. EYE OF A VOLUTE FROM THE TEMPLE OF ARTEMIS AT EPHESES	58
FIGURE 66. EYE OF A VOLUTE FROM PRIENE	58
FIGURE 89. THE DORIC TEMPLE OF APOLLO EPIKOURIOS AT BASSÆ	78
FIGURE 96. THEATRE OF DIONYSOS. PATTERN IN THE FLOOR OF THE ORCHESTRA	88
FIGURE 100. THE INTERIOR OF THE CELLA OF THE PARTHENON	96
FIGURE 101. THE NORTHEAST CORNER OF THE PARTHENON STEPS	98
FIGURE 102. STEPS AT THE WEST END OF THE PARTHENON . .	100
FIGURE 103. PRESENT CONDITION OF THE SOUTHEAST CORNER OF THE CELLA	100

PREFACE

IN this book Mr. Hambidge presents in detail the evidence on which he bases his theory that the Parthenon was designed in accordance with the principles of proportion called by him "Dynamic Symmetry." Historians of Greek art who are interested in such investigations are now enabled to judge whether his results are "geometrically exact," and to estimate the probabilities for or against their being also "historically true." Mr. Hambidge's analyses are based on the careful measurements of Penrose, checked and supplemented by measurements made by himself. Since the arithmetical proof that the dimensions of the building accurately fit the ratios of his system is fully stated in the appendix, the reader who is willing to take the trouble can answer the first question for himself. The second question, on the other hand, is one which, in the absence of clear literary tradition, can never receive a final answer. We do not even know whether the architects of the Parthenon used mathematical proportions in designing it, although this appears to be regarded as not improbable by such modern writers on Greek architecture as have touched upon the problem. And, assuming that some system was used, we have no clue which can help us to rediscover it. For the precepts of Vitruvius, though they are believed to echo, obscurely and faultily, rules of proportion which had actually been worked out and used by Greek architects, have never been shown to fit even Ionic buildings of the fourth century or later, and are manifestly inapplicable to the Doric temples of the Periclean age. The only way to approach a solution of the problem is to apply various possible systems to an existing building, such as the Parthenon, and to judge whether a strong circumstantial case can be made out in favor of any of them.

Many attempts have been made in the past, but unfortunately with results which seem to have convinced few save their authors. I should like here to call attention briefly to two of the methods of analysis which have been tried, in order to contrast them with the new attempt set forth in this book. A study of the proportions of the Parthenon by W. Watkiss Lloyd was published in the second edition of *The Principles of Athenian Architecture*, with a statement by Penrose that it completely solved the problem. Lloyd worked with numerical ratios. Realizing that the possible ratios between 1:1 and 1:10 are infinite, he limited himself to two series with low numbers, and claimed that the architect of the Parthenon used (1) a series in which the second number is greater than the first by five units, *i.e.*, 1:6, 2:7, 3:8, 4:9, 5:10, etc., and (2) the simple series 1:2,

2 : 3, 3 : 4, 4 : 5, etc. But, though he found in the building certain striking instances of some of these proportions, his theory has been little regarded; nor has his very incomplete investigation been carried farther. The number of elements in the Parthenon which can be so compared is very great; more or less accurate coincidences with these ratios are bound to occur, as are also coincidences with ratios not belonging to these series (*e.g.*, the abacus width of the corner column is to the height of the column as 1 : 5). It is noteworthy also that in his analyses of two other temples (the Theseum and the temple at Bassæ) Lloyd did not confine himself to these two series. Nor did he succeed in showing any logical relationship between the proportions which he noted. The ratio 4 : 9, to which the stylobate of the temple very nearly conforms, is an easily comprehended arithmetical concept. But its æsthetic significance and its practical usefulness in the design of the temple remain obscure. In short, the study of Lloyd's system does not reveal any coördinating principle which might be regarded as giving the ratios he uses an æsthetic or practical value.

Another method of analysing the proportions of a Greek temple is to draw a geometrical figure—say, a hexagon or a six-pointed star—over a plan of the building, and then to show that important divisions of the plan coincide with intersections of lines connecting important points on the geometrical figure. It is obvious that the more lines you draw the more coincidences you will get. Such operations are unworthy of serious examination. One of the weaknesses of this graphic method has been pointed out by Chipiez. The geometrical figure is constructed, with more or less inaccuracy, on a more or less inaccurately drawn small-scale plan of a temple, the dimensions of which may have been more or less inaccurately measured. Seeming coincidences between the plan and the geometrical figure often turn out on investigation not to be coincidences at all. No analysis of the proportions of a building is worthy of study unless it is based on accurate arithmetical calculations.

In recent years archæologists have turned from such speculations to careful study of the actual remains, whether of buildings previously known or of new material brought to light by excavations. They have studied proportions from the historical point of view; that is, they have compared the proportions found in buildings of one period with those found in buildings of earlier and later periods. They have also ascertained the length of the Greek foot, and have sought to express the dimensions

of architectural members in terms of the foot used. But, in doing this, they can hardly hope to do more than rewrite the specifications of the Greek architects. They can throw no light on the symmetry, *i.e.*, the harmony of proportions, inherent in an architectural masterpiece.

In the circumstances it is natural that archæologists should look with suspicion upon any new theory of a mathematical symmetry underlying the Parthenon. And Mr. Hambidge's theory, by reason of the very fact that it conforms to early Greek mathematical thought, is difficult of comprehension by us of the present day. We think arithmetically rather than geometrically. We can readily comprehend the proportion 4:9; but the attempt to understand the simple proportional properties of the root-five rectangle seems to cause many people acute mental anguish.

Symmetry is most simply defined as commensurability. To us this means that two dimensions to be compared can be measured in multiples of a common unit. Mr. Hambidge has called attention to the fact, historically established but generally overlooked, that the Greeks recognized a geometrical, as well as an arithmetical, symmetry. Two dimensions, though not commensurable in simple linear units, may be simply commensurable in square—*δυνάμει σύμμετροι*, to use the Greek term occurring (as Mr. Hambidge has noted) in the earliest extant mathematical record; that is, "commensurable in power," or, one might say, "dynamically symmetrical." The name which Mr. Hambidge has given to his theory of proportions based on the "root rectangles" turns out to be a transliteration of a Greek term expressing the same idea. That the Greeks of the fifth century were familiar with the root rectangles is clear from the well-known passage in the *Theætetus* of Plato. Eudoxus, a contemporary of Plato, is said to have investigated "the section," which is held to be the *sectio aurea*, *i.e.*, the principle of extreme and mean proportion inherent in the root-five rectangle. These proportions also underlie much of the geometry of Euclid.

It is surprising that critics of Mr. Hambidge's theory have passed over in silence his claim that it has an historic basis. They might argue that artists are not mathematicians, and that the constructions of dynamic symmetry are too complex to have been comprehended at all by Greek artisans, or to have been put to a practical use even by architects such as Iktinos. But the science of geometry is held to have been developed gradually from simple methods of land measurement. The proportions

found by Mr. Hambidge in the Parthenon could have been laid out with the help of pegs and a cord almost as easily as the construction which builders have used from time immemorial to establish a right angle. It is historically recorded that sacrificial altars were constructed in ancient India in accordance with the proportions of the root rectangles. What the Hindus actually did, Greek architects surely were capable of doing. It is a mistake to assume that a geometrical construction is abstruse because it

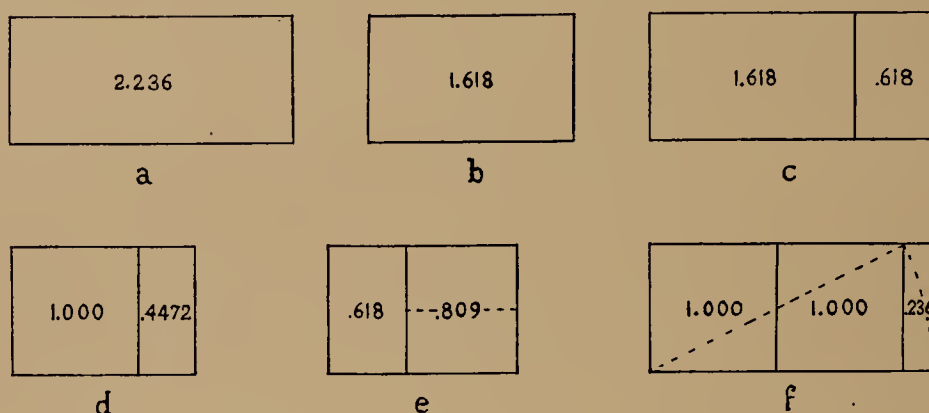


Figure A.

is possible to describe it in abstruse language. For example, Chipiez cites the plan of the temple of Concord at Girgenti as one which suggests a mathematical proportion. Its length, he says, is "exactly equal to four times the side of the regular decagon inscribed in a circle whose radius would be equal to the width of the façade" (Perrot et Chipiez, *Histoire de l'art antique*, VII, p. 558). This sounds very formidable. But the geometrical construction by which such a temple plan could have been laid out (*ibid.*, p. 556, fig. 252, IV) is so simple that he finds it unnecessary to explain it. "It is not impossible," he adds, "that this result, geometrically exact, may also be historically true."

Mr. Hambidge's critics have had but one tangible argument to advance against his theory. It is that the ratios, or rectangles, of his system are innumerable. Consequently any object under the sun could be analysed in accordance with it. What the critics have failed to explain is the constant recurrence in Greek works of art of a very limited number of proportions derived by simple constructions from the square. Readers of this book will find that the following proportions occur over and over again in the Parthenon:

- (1) The root-five rectangle. Ratio, 2.236. Reciprocal, .4472. Fig. A, a.
 (2) The extreme and mean proportion rectangle. Ratio, 1.618. Reciprocal, .618. Fig. A, b.

This, with its reciprocal added, equals the root-five rectangle, Fig. A, c.

- (3) The rectangle composed of a square and a root-five rectangle. Ratio, 1.4472. Reciprocal, .691. Fig. A, d.

- (4) A combination of a .618 rectangle with half of a 1.618 rectangle. Ratio, $.618 + .809 = 1.427$. Fig. A, e.

- (5) The rectangle which remains when two squares are subtracted from a root-five rectangle. Ratio, $2.236 - 2.000 = .236$. Fig. A, f.

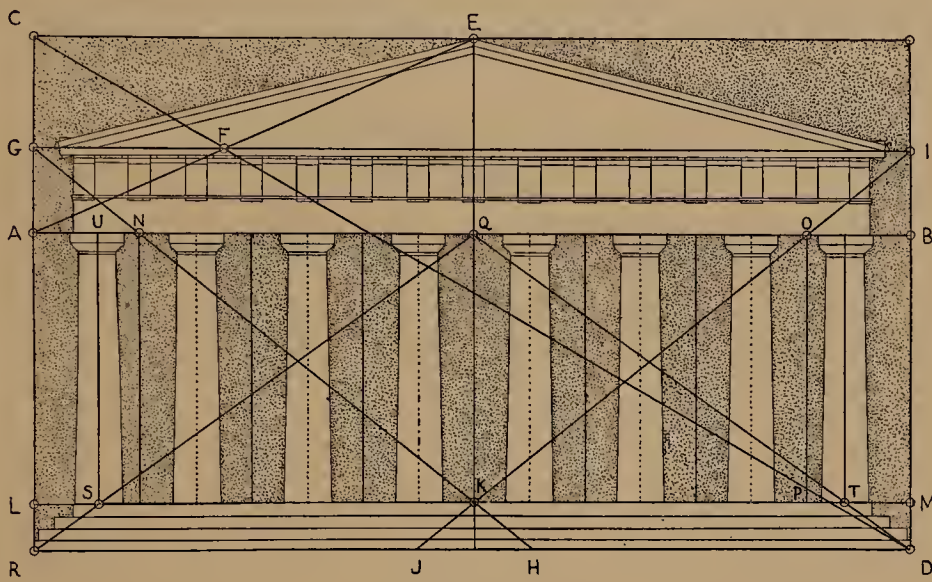


Figure B.

As a clear illustration of Mr. Hambidge's method I should like to call attention especially to his analysis of the façade of the temple. The following is a summary, or restatement, of his second chapter. The essential proportions, which he has explained by numerous diagrams, are shown in the above drawing, figure B, in a single geometrical figure superimposed upon an elevation of the façade.

It will be well to consider first what are the most important proportions to be determined.

The most logical procedure is to begin with the relation of the greatest height to the greatest width. The top of the enclosing rectangle is given by the apex of the pediment, including the sima. But a problem arises in determining the bottom of the rectangle and its width. The temple rests

on a base of three steps above a fourth, slightly projecting step, called by the Greeks *εὐθυνηρία*, or levelling course. In the Parthenon this course is built of marble on the west and north sides, of limestone on the south, while, on a portion of the east side, the native rock takes its place. It has been argued that this course, since it varies in thickness as well as material, should be excluded from an analysis of the proportions. But the *εὐθυνηρία* is a normal feature in Greek temples. It was given a special name; and this name suggests its function as a guide for the laying of the superstructure for which it served as a frame. Since the Parthenon was designed "on paper" before it was built, and since a study of its proportions must be concerned with the original design, and not with departures from it due to special circumstances, it seems logical to include the levelling course in the proportional scheme.

Within this rectangle what are the most important horizontal and vertical lines of the façade? Everyone will agree that there are three main horizontal divisions:

(1) The line of the top of the entablature, the base of the pediment triangle.

(2) The line of the tops of the columns.

(3) The line of the stylobate.

The façade divides logically into:

A. Gable,

B. Entablature,

C. Columns,

D. Base.

Of the three horizontal lines the most important is undoubtedly the division between the supporting members (columns) and their load (entablature and pediment).

The most important vertical lines are the axes of the eight columns. Since in a Doric temple the intercolumniations next to the angles are less than the others, it becomes necessary to determine:

(4) The axes of the six intermediate columns.

(5) The axes of the angle columns.

Here I should like to observe that the spacing of the columns should be considered with reference to the elevation, not, as has usually been done, with reference to the ground plan. The columns are seen as vertical supports, not as rows of circles on a floor. Their spacing must be brought into

harmony with their height and with other proportions of the façade. The usual procedure is doubtless due to two causes: (1) The influence of the precepts of Vitruvius, and (2) the fact that many temples survive only in plan, and that ground plans rather than elevations are studied by archæologists. Undue importance has been attached to the lower diameter of the columns, which is often taken as a module, or unit, for establishing other proportions.

Let us now consider Mr. Hambidge's analysis.

The containing rectangle has the ratio 1 : 1.708, or 1 : $3\sqrt{5} - 5$. It may be regarded as composed of the elements shown in the small diagram, figure C, which also makes clear the proportional relationship between the façade and the ground plan. Thus:

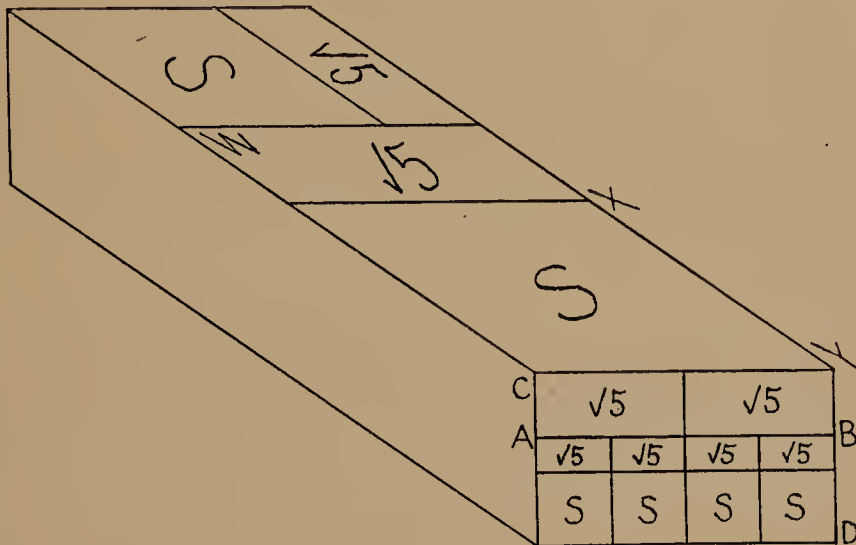


Figure C.

$$CB = \frac{1}{2}WX. \quad AD = \frac{1}{4}WY.$$

The rectangle of the façade may also be subdivided into the simple elements shown in figure D.

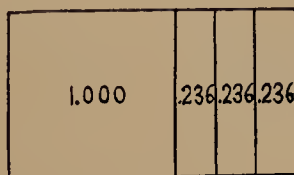


Figure D.

We next examine the five secondary proportions in the order given above:

(1) The level of the top of the entablature. This is seen to coincide with the intersection at F of two diagonals—(1) that of the enclosing rectangle, CD, and (2) that of one of the upper root-five rectangles.

(2) The most important horizontal division—the line of the tops of the columns, AB, coincides with the main division of the enclosing rectangle. The entablature and pediment are thus contained in an area made up of

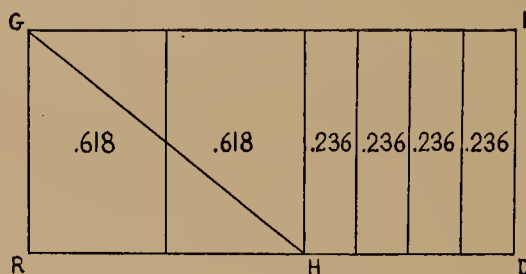


Figure E.

two root-five rectangles, the steps and columns in an area made up of four squares plus four root-five rectangles.

It is noteworthy that this important line, AB, cuts the complete height of the temple in extreme and mean ratio.

(3) The level of the stylobate. The area below the pediment base line, GD, may be expressed graphically as in figure E. That is, it is composed of two .618 areas plus four .236 areas. In the drawing of the façade, figure B, the line GH is a diagonal to two .618 areas. The line IJ is a similar diagonal. The point of intersection of these two diagonals, K, is at the level of the stylobate. These two diagonals show further that the columns and entablature considered separately are contained in an area, GM, made up of four .618 rectangles. $.618 \times 4 = 2.472$, or $2\sqrt{5} - 2$. This is the proportion mentioned by Chipiez, which Mr. Hambidge has found to be that of the ground plan of the temple at Bassæ.

(4) The axes of the six intermediate columns. The diagonals, GH, IJ, cut the line of the tops of the columns at N and O. These points are on the axes of the second triglyph from each corner, and thus have a relation to the spacing of the six normal columns. The area NP, being similar to GM, is composed of four .618 rectangles. Since there are six intermediate columns, this area must be divided into six parts, each of which will be

composed of .618 rectangles, arranged as in figure F. A column is centered in each of these six rectangles.

It is noteworthy that this column-centering rectangle appears to account for the proportions of the capitals (see Chapter Four).

.618	.618
.618	.618
.618	.618

Figure F.

(5) The axes of the angle columns coincide with the points of intersection of the diagonals QR, QD, with the stylobate line at S and T.

The area UT, being similar to AD, is made up of four squares plus four root-five rectangles.

The analysis may be described in the briefest terms as follows:

The main proportions of the façade are shown in a rectangle closely related to that of the ground plan. The vertical and horizontal subdivisions of this rectangle are given by the members of the temple themselves. The proportional relationship of these members is shown by six diagonal lines.

If Penrose's measurements are postulated to be correct, the demonstration has the character of a proposition in Euclid. One is impelled to subscribe Q. E. D. The result may be explained as due to design on the part of the architect, or to a combination of fortuitous coincidences. The phenomenon is striking enough to deserve investigation.

The use of such a geometrical scheme by the architect would have two practical advantages:

(1) Models of the whole building or of any of its parts could be con-

structed at any scale, without mathematical computations, and with the certainty that they would be absolutely accurate as far as the proportions were concerned.

(2) The geometrical figure could be laid out full size on the ground; and, with the help of a measuring tape, the actual dimensions of every member could be accurately ascertained and recorded in Greek feet and their subdivisions. It is difficult to imagine a simpler method of making such a list of dimensions, which was indispensable for writing the specifications for the quarrying, finishing, and setting in place of the blocks of marble.

The purpose of this brief discussion has been to bring out four points which, it seems to me, should be considered by readers of this book. They are:

(1) Does the theory conform to mathematical principles of proportion known to have been familiar to the Greeks of the fifth century B.C.?

(2) Are these principles simple enough to have been intelligible to artists as well as to mathematicians? Could the geometrical figures be easily constructed with ruler and compasses, or with strings stretched between pegs driven into the ground, and without mathematical computations?

(3) Do the most important dimensions of the Parthenon—especially those measured in tens and hundreds of feet—conform accurately to these geometrical figures?

(4) Would the employment of these geometrical figures be of practical aid to the architect in carrying out his design?

I might add, as a fifth point, the question whether the use of such a system might conceivably contribute toward producing that harmony of proportions which has always been admired as the supreme quality of the Parthenon. But I prefer to leave to others the discussion of Dynamic Symmetry in its æsthetic aspect.

If the four questions I have raised receive an affirmative answer, it does not follow inevitably that the architects of the Parthenon used the system discovered by Mr. Hambidge. But I think the claim can fairly be made that he has investigated an interesting and very difficult problem according to a logical and scientific method. And it is on these grounds that the Yale University Press has undertaken the publication of the book.

L. D. CASKEY.

INTRODUCTION

DYNAMIC SYMMETRY is an ancient Greek term, *δυνάμει σύμμετρος*, meaning *commensurable in power*, particularly the second power or square of a number. The term was used to describe the lines which constitute the sides of rectangles bearing a square-root relationship with their ends. These side and end lines are incommensurable (asymmetrical), but the squares on them are commensurable (symmetrical), as 1:2:3:4:5, etc.

The term seems to have been an invention of the early Greeks as it appears in the most ancient fragment of their geometry which has survived. For many hundred years it was used to distinguish the lines mentioned. (See Note I.)

See Euclid X, Def. 2.

Diophantus of Alexandria (fl. A.D. 250) was the first to make use of algebraic symbols. For the powers of the unknown he used $\Delta\gamma$ for *δύναμις*, the square, and $\kappa\gamma$ for *κύβος*, the cube, and so on.

The analyses of the proportions of the early Greek temples are based upon one of the rectangles of the classification mentioned—that with end of unity and sides equal to the square root of five. It should be remembered that this root-five line is merely a diagonal to two squares.

By a logical process this rectangle is subdivided to produce other rectangles and these, with the generating area, explain the proportions of the plans of the temples in two and three dimensions.

The numerical value of the square root of five is 2.236+. The fraction is endless.

The rectangles obtained by subdividing a root-five area are:

.236, which represents the difference between two squares (2) and 2.236. This fraction is the reciprocal of 4.236. (.236 equals root five minus 2; 4.236 equals root five plus 2; .618 equals root five minus 1, divided by 2; 1.618 equals root five plus 1, divided by 2.)

.618. This fraction is the reciprocal of 1.618.

The last represents the celebrated extreme and mean ratio of the Greeks which was so well known by the time of Plato that it was called "The Section."* In more recent times it was called "The Divine Section" and still later "The Golden Section."

* "We are told by Proclus that Eudoxus 'greatly added to the number of the theorems which Plato originated regarding *the section*, and employed in them the method of analysis.' It is obvious that *the section* was some particular section which by the time of Plato had assumed great importance, and the one section of which this can safely be

The temples which have been analysed in the light of dynamic symmetry are:

The Parthenon, which forms the bulk of this book.

The temple of Apollo at Bassæ in Arcadia by Iktinos, the architect of the Parthenon.

The Zeus temple at Olympia.

The temple at Ægina.

The temple at Sunium near Athens.

Considerable work has been done on the early Greek temples in Sicily and elsewhere, but this material is not ready for publication.

In a previous volume, *Dynamic Symmetry: The Greek Vase*, and in the magazine *The Diagonal*, the small monthly journal containing disconnected material which came to light while the investigation was in progress (published in twelve numbers during 1920-1921), the difference between dynamic and static symmetry was sketched. Therefore it will not be necessary to repeat the distinction. Reference, however, may be made to the architectural plans by Michael Angelo now in the Buonarrotti museum at Florence, Italy, and the sketch-books of Leonardo da Vinci and Villars de Honnecort for illustrations of the static type of symmetry used during the middle ages and renaissance.

The static symmetry used by the Romans is very well exemplified by the architectural principles enunciated by Vitruvius. This author's modulus scheme would result automatically in this lower grade of symmetry. He does recommend a root-two proportion for an atrium but apparently did not understand anything of its properties—to him it was probably an echo of a tradition. The fact that the Roman writer supplies us with elaborate explanations of the proportions of Greek buildings and that none of them are verified by the actual remains is rather conclusive proof, it seems to me, that he did not know what they were.

Vitruvius lived in the time of Augustus and the Greek authorities he mentions flourished during the decaying period of classic art when the principles observable in the early architecture had become lost.

During Nero's time, shortly after Vitruvius, the theatre of Dionysos at Athens was rebuilt and arranged for certain barbaric spectacles. The

said is that which was called the 'golden section,' namely, the division of a line in extreme and mean ratio which appears in Euclid 2 : II, and is therefore most probably Pythagorean." *The Thirteen Books of Euclid's Elements*, T. L. Heath, Vol. I, p. 137.

new floor of the orchestra was then decorated with a large pattern made by pavement blocks. This floor is in a fairly good state of preservation, as the photograph shows. (Note II and figure 96.) Its proportions are easily ascertainable and they probably shed as much light upon Roman design thought as anything which has survived. This pattern is just the sort of thing we should expect from the Vitruvian principles, but it is far away from the design of ancient Greece.

The outstanding proportions of this pattern are root three to one (Note II), but, like the root-two proportions of the Roman atrium, there is no indication that its designers knew anything of the properties of the rectangle. I am inclined to believe that the designers of this period were trying to use an old Greek method, the meaning of which had become obscure. The early Greeks used the root-three proportion correctly to a limited extent in certain types of design but not in architecture, if the Choric Monument of Lysicrates is excepted. To a certain extent therefore we have positive evidence of the use of root rectangles in Roman design much in the same manner that these figures were employed by the artists of the middle ages.

There are a few surviving grave stelæ which throw some light upon the question of the date for the appearance of dynamic symmetry in Greece. One is now in the Museum at Athens, one in the Metropolitan Museum of Art, New York, and one in the Museum of Fine Arts, Boston. The two former seem to have been made at Athens about 550 B.C. The one at Boston is from the Troad and its date is probably about 500 B.C. These three stelæ are examples of dynamic symmetry design.

The example in the Museum at Athens discloses such a simple scheme of dynamic symmetry that it furnishes an admirable illustration of this type of proportion. The measurements, made by well-known authorities, and the calculated proportions are shown in the table.

	<i>Measurements.</i>	<i>Calculation.</i>
Width of abacus	0.678 meters.	0.678 meters.
Width of neck	0.420 meters.	0.41903 meters.
Height of abacus	0.129 meters.	0.12949 meters.
Depth of abacus	0.255 meters.	0.2554 meters.
Depth of neck	0.170 meters.	0.1695 meters.

The difference between the symmetry scheme and actual measurement is less than a millimeter for all the details.

The width of the abacus is used as a side of a square to reduce the two dimensional plan to terms of area as is done with all the Greek temples. The symmetry scheme is begun by placing a .618 rectangle in the center of this square. This first step, however, finishes the process as all the proportions of the design are at once established.

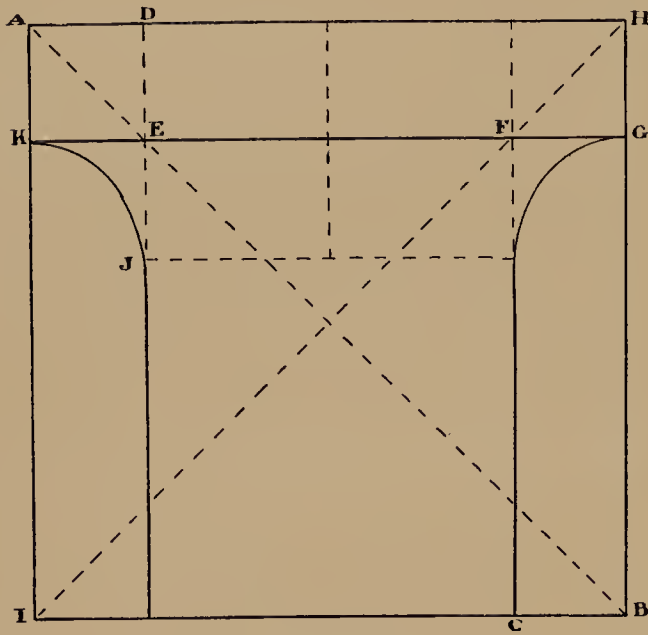


Figure 1.

Diagram of the Grave Stele at Athens.



Figure 2.

Grave Stele at Athens.

AB is the square fixed by the abacus width.

CD is the .618 rectangle.

AE, HF are squares.

Through E and F the line KG fixes the abacus AG.

CE is composed of the square CJ and the two .618 areas JF.

DF is composed of two .618 rectangles.

AG, DI, and CH are similar and equal figures, each being composed of a root-five rectangle and three squares.

IE and BF are each composed of a root-five rectangle and two squares.

(The line IB is very close to a moulding on the original, therefore it may be that the plan was intended to be a square.)

It will be noticed that the voids as well as the solids have a proportion value. This is as it should be in a graphic proportioning scheme which

fixes symmetry by area and volume rather than by line. The symmetry of the Greek temples is fixed precisely as in this diagram and the proportions of this stele are the actual proportions found in the temples. The entire scheme of dynamic symmetry may be learned from this single example. We might extend the inspection of the stele to the side elevation and thus obtain the three dimensional relations, or fix the plan of a section say at IB (actually a similar figure to the great column-spacing rectangle of the façades of the Parthenon), or the plan of the top at AH, but this is

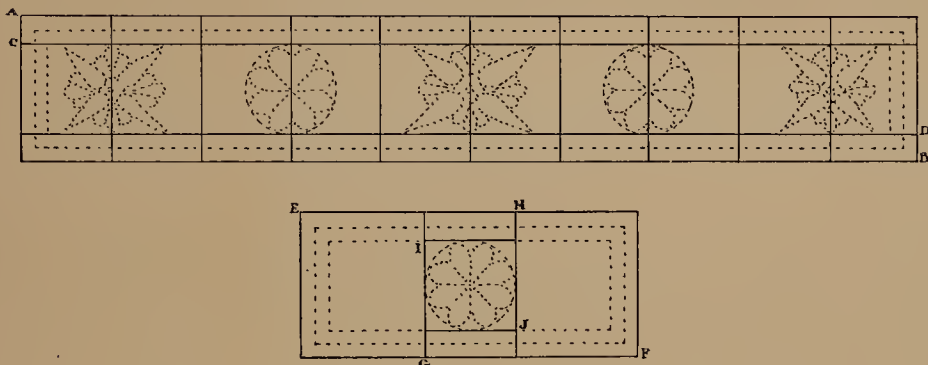


Figure 3.

Decoration on the Abacus of the Stele at Athens.

not necessary for the present purpose as more pertinent material is available in abundance. (See Note III for pottery proportions.)

The abacus of the stele is decorated with a pattern. The arrangement of the units of this pattern is such that they echo the proportions of the design as a whole. The area AB of the abacus face is composed of ten .618 rectangles, while that of CD, the height of which is fixed by the decorative units, is composed of ten squares. Thus each .618 rectangle has a square in its center. The error of this arrangement is extremely small.*

The area of the end is shown by EF, of which HG is a .618 rectangle. The areas EG and HF, together, represent a similar figure to a façade

* An attempt has been made to analyse this stele in terms of static symmetry (see *American Journal of Archaeology*, July-September, 1922, pp. 261-277), but as the errors are so considerable—nearly a centimeter at important points in so small an example—the evidence points against rather than for its use.

projection of the Parthenon. It is interesting to note that the area of the façade of the abacus, ten .618 rectangles, appears in the ground plan scheme of the Erechtheum.

The use of two measuring systems, the English and the metric, in the temple analyses may cause some confusion, but I believe it is unavoidable as the most reliable measurements of the Parthenon are those made by the English architect, Francis Cranmer Penrose. He, however, appreciated the weakness of the English system for the purpose and had the foresight to employ a decimal division of the foot. I have had occasion to check Penrose's work and, except in some minor particulars, have found it very accurate.

A peculiarity of the plans of Greek temples of the classic period is the unequal spacing of the front and flank columns. This applies especially to the temples of Greece proper and not so much to those of the colonies. This inequality of spacing, I am inclined to believe, is a sure indication of the use of dynamic symmetry by the ancient architects. If static rectangles were used for planning, the most logical process of subdivision would be in even multiples for fronts and flanks. If no rectangles were employed and the plan and its subdivisions were fixed by length units, whether by a modulus or a foot or cubit, a static result would follow automatically. Using the foot or the meter the modern architect almost invariably makes equal spacing around a building, and so did the architects of the Hellenistic period.

Another peculiarity, of similar character to the irregular column spacing in the classic Greek building, is that of inequality of step heights and treads. Again the modern architect makes these equal, as did the Hellenistic Greek designer.

This is logical if static design is employed, but more or less illogical if a dynamic method of setting out the plan is used. The ends and sides of dynamic rectangles can hardly be divided in equal multiples. If they could be so divided then the entire interior of the plan would be arranged in squares by lines connecting the column centers.

Thus it is clear that whatever explanation is made of classic Greek methods of fixing temple plans the fact of front and flank differences in column spacing and inequalities of step heights and treads must be considered. Up to the present I believe dynamic symmetry offers the only rational explanation of these peculiarities.

CHAPTER ONE: "THE GREEK TEMPLE"

HOW THE BUILDING IS ANALYSED

HERE are two methods of analysis which may be used for a study of the character of the proportions of the Parthenon at Athens.

One employs certain subdivisions of the temple plan which were fixed by the architects, such as areas defined by column centers.

The other consists of the fixing of the proportional character of voids and solids as made by the two or three dimensional plans and elevations.

The second method is sweeping in character and discloses every possible phase of proportional or non-proportional relationship.

The first method is necessarily limited to certain parts of the two dimensional plan; the second embraces every aspect of two and three dimensions.

As was explained in the Introduction there are three rectangles used in the analysis of the temple.

(1) A root-five rectangle with the value of 2.236, reciprocal .4472.

(2) A 1.618 rectangle, reciprocal .618.

(3) A .236 rectangle, which represents the difference between two squares and a root-five area (2. and 2.236). The fraction .236 is the reciprocal of 4.236, that is, root five plus two squares.

When lines are drawn across the floor of the temple from center to center of the regularly spaced columns of the façades, as they are fixed by the centers of the angle columns, the area thus obtained is composed of .618 rectangles arranged in a peculiar manner. The angle columns are larger in diameter than the intermediate columns; hence, the centers of the former are slightly farther away from the edge of the top step than the latter.

There are six regularly spaced columns on each front, consequently the area made by the six strips divides the major part of the temple floor into a trellis of .618 rectangles. (See figure 4.)

A, B, C, D, E, F are centers of the six regularly spaced columns of a façade as fixed by the centers of the angle columns.

GH is composed of two .618 rectangles.

HI is a .618 area. There are eleven of these groups in each of the six strips.

When lines are drawn across the temple floor from center to center of the flank columns, fixed in the same manner as are those of the fronts, the

result is a series of area strips each of which is composed of .236 rectangles and squares. (See figure 5.)

AD is a strip fixed by the column centers of the flanks. EF, FG are 2.36 rectangles (note the position of the decimal point), each of which is a similar figure to the column center rectangle of the cella.

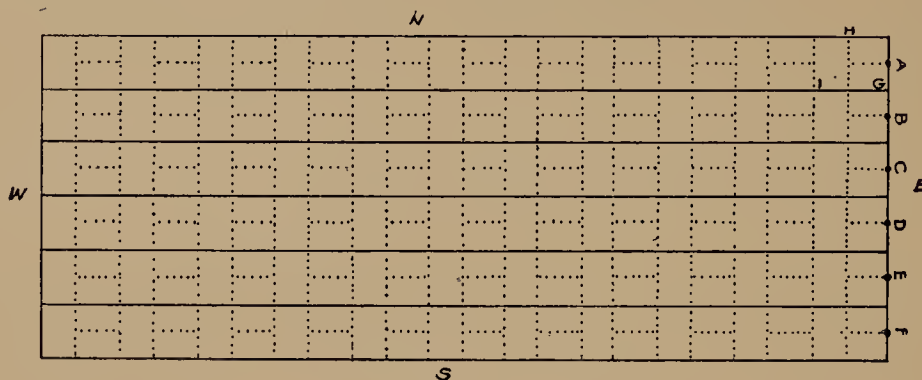


Figure 4.

BE, GD are squares. There are 15 of these strips.

Figures 4 and 5 solve a difficult problem, namely, that of the subtle difference between the widths of the front and flank intercolumniations. (See Appendix, Note IV.)

The centers of the regularly spaced columns of the fronts have a mean separation of $14.1+$ feet. (See Appendix, Note V.)

The mean for the flanks is 14.086.

The difference between these two spacings might be thought negligible by the casual reader but the amount is considerable in a length of a couple of hundred feet.

The areas of these two strips are essentially dynamic in their proportions and are part of the design scheme of the building.



Figure 5.

For the second method of analysis it is necessary to reduce the outstanding proportions of the structure to rectangles and to regard these as subdivided into other rectangles by architectural members such as columns, entablature, pediment or steps by their heights and widths on fronts or flanks. These again may be subdivided into smaller areas until each separate architectural unit is bounded by its own particular rectangle.

The full height and width of the building, for example, furnish an end and side of a rectangle.

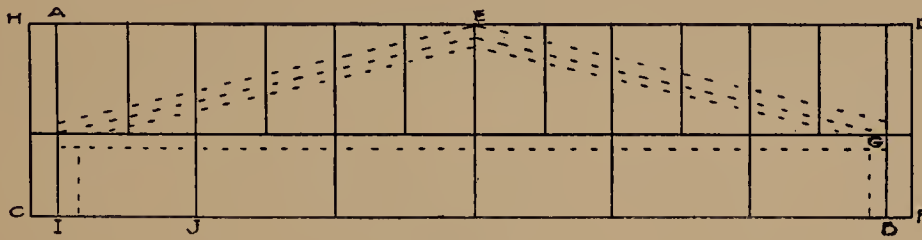


Figure 6.

The full width of a façade is provided by the full front length of the lowest step. Other members of the façade, however, such as entablature and pediment, are shorter than the lowest step; hence, there will be voids as well as solids represented within the rectangle, but the parts of the overall rectangle occupied by these voids must have the same proportional character as the rest. The analysis will, therefore, be an exhaustion of the major rectangle by the void and solid projections. The process may be carried to any degree of refinement necessary.

Figure 6 is an illustration of the second method.

The full length of the entablature, with its corona and mouldings, and the height of the entablature and pediment combined, provide a rectangle composed of .618 rectangles arranged like those of figure 4 (AJ is similar to one of the groups of figure 4.)

The cornice is shorter than the lowest step by an amount shown by the width of the rectangles AC and BD. These two small areas are composed of .618 and .236 rectangles.

The major rectangle of the diagram, HF, is composed of two root-five rectangles, CE and EF.

Hence the entablature and pediment projection provide a rectangle, AB, within a double root-five rectangle HF, and the difference between the two is represented by the areas AC and BD.

THE GROUND PLAN

THE ground plan of the Parthenon is fixed by its lowest step. This is the first levelling course, known as the *euthynteria*. The width of the end of this rectangle is 111.31, the length of a side is $238.003 \pm$ English feet. These measurements are demanded by the trellis scheme which has been found in the plan. They differ from the direct measurements obtained by experts by less than one-quarter of an inch in the length. The width is exact. One side of the building is slightly longer than the other, however, by an amount which totally eliminates the error.

The area of the ground plan rectangle is composed of two squares and two root-five rectangles arranged as in figure 7.

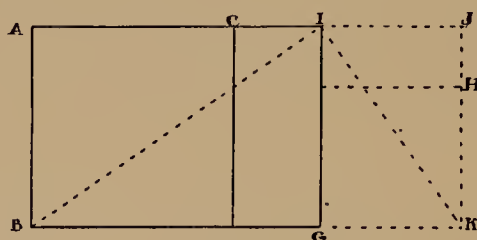


Figure 7.

BC, GH are squares, while CG, IH are root-five rectangles.

The square and root-five area IK is fixed by the right-angled triangle BIK.

THE FAÇADE

PENROSE measured and calculated the height of the Parthenon from the ground to the top of the sima. He was not able to measure all the members as the roof and the upper parts of both pediments were destroyed.

The following is a comparison of the actual measurements with those demanded by the pattern trellis. (Measurements in feet and thousandths.)

	<i>Penrose.</i>	<i>Dynamic Trellis.</i>
Steps	6.058	6.025
Column (angle)	34.253	34.247
Entablature	10.793	10.779
Pediment	14.079 (calculated)	14.111
<i>Total</i>	<u>65.183</u>	<u>65.162</u>

The greatest difference is that of the pediment height, which Penrose was unable to measure but assumed that it was equal to one intercolumniation. This height does suggest a façade intercolumniation, and the dynamic figures are much closer than those of Penrose, but I leave the question open.

The column heights are for those at the angles. The middle ones are shorter by something less than half an inch. This difference is due to the fact that the horizontal curvature of the epistyle is less than that of the stylobate upon which the columns stand.

The most striking aspect of the façade rectangle, fixed at 65.162 by 111.31 feet, is that it is composed of a square and three .236 rectangles.

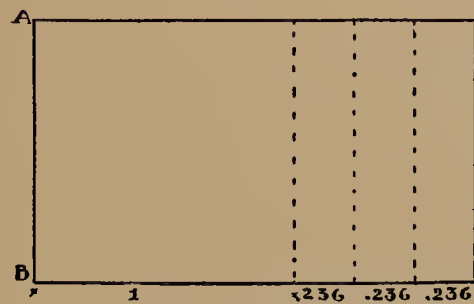


Figure 8.

Another interesting feature is that the bottom of the epistyle, or top of the columns, divides the area of the façade projection in extreme and mean ratio. AB is thus divided at C (figure 9).

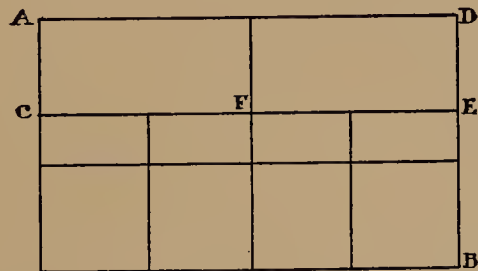


Figure 9.

The bottom of the epistyle, the line CE of the diagram, divides the façade rectangle into squares and root-five areas.

AE is composed of two root-five rectangles, AF, FD.

CB is composed of four squares and four root-five rectangles.

The diagram is a comparison of the rectangles of the façade and ground plan by imposing one rectangle on the other.

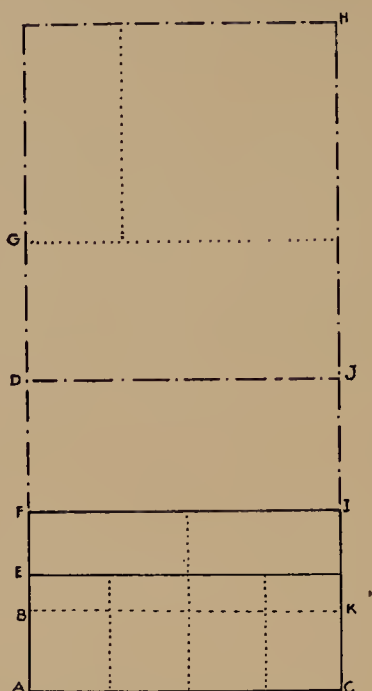


Figure 10.

AH is the ground plan area.

AJ is the ground plan square.

AI is the façade rectangle.

AK are the four squares of the façade.

AB is one-fourth of AD.

AE is one-fourth of AG.

EF is one-half of DG.

BE is one-half of EF.

GH is the reciprocal of CG.

The bottom line of the epistyle, probably the most noticeable division of the building, brings into prominence the celebrated Greek proportion referred to by the ancient authorities as "The Section." This line divides the elevation of the structure into two parts on fronts and flanks which, individually and collectively, bear to each other this extreme and mean relationship. The area AE of figure 9 is to the area CB as is .618 to 1.

The area CB of figure 11 bears the same relation to AD, etc.

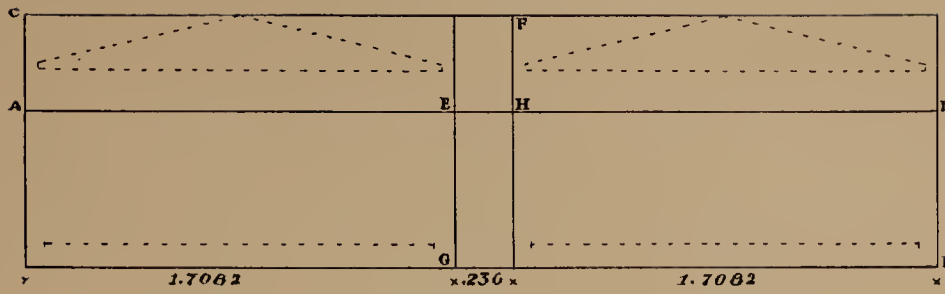


Figure 11.

The flank elevation of the Parthenon. Equal to two façades plus a .236 figure.

THE FLANK ELEVATION

THE flank elevation, 65.162 by 238.003 feet, furnishes an area composed of two rectangles of the façade plus a .236 figure (figure 11).

CG and FD are each equal to a façade rectangle.

GF is a .236 rectangle.

CE and FB are each composed of two root-five rectangles.

FE is a .618 rectangle.

AG and HD are each composed of four squares and four root-five areas.

GH is composed of a 1.618 area plus a square.

As the method of analysis is to begin with the outstanding proportions, consideration of details follows in succeeding chapters.

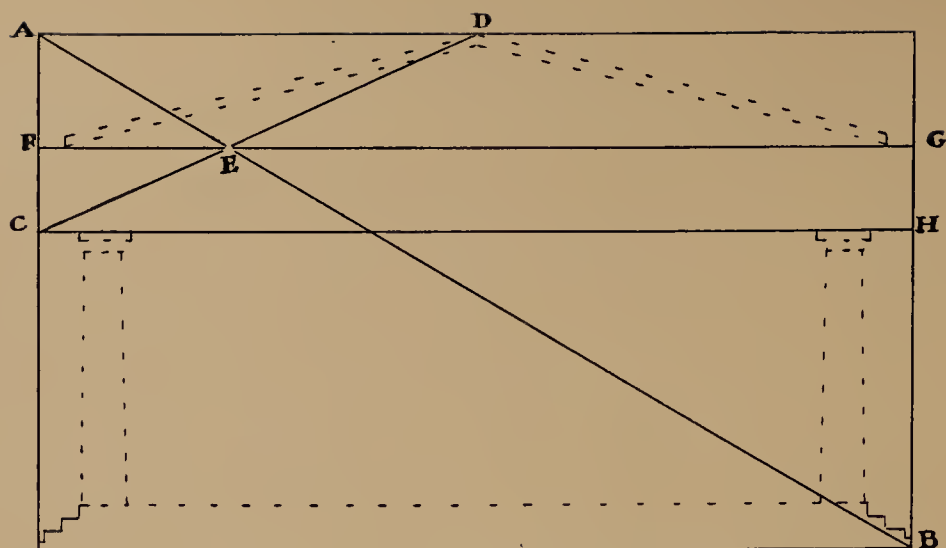


Figure 12.

CHAPTER TWO: THE FAÇADE AND ITS DIVISIONS

AB (figure 12) is the rectangle of the façade.

CH is the top of the columns.

CD, DH are root-five rectangles.

(See figure 9 for façade construction.)

AB is a diagonal to the entire rectangle; CD a diagonal to a root-five area.

These lines intersect at E.

FG, drawn through E, is the floor of the pediment or the top of the entablature cornice.

AG is the rectangle for the entire pediment projection while FH is that for the entablature.

The area AG is readily analysed by the diagonal lines AE, ED.

FD is half of the area AG.

AE is a similar figure to AB while ED is a root-five rectangle.

Thus AG is composed of two similar figures to a façade rectangle plus two root-five rectangles.

FE of figure 13 represents the area of the complete entablature projection.

It is composed of four root-five rectangles plus the area HI.

This latter area is composed of two squares and two root-five rectangles.

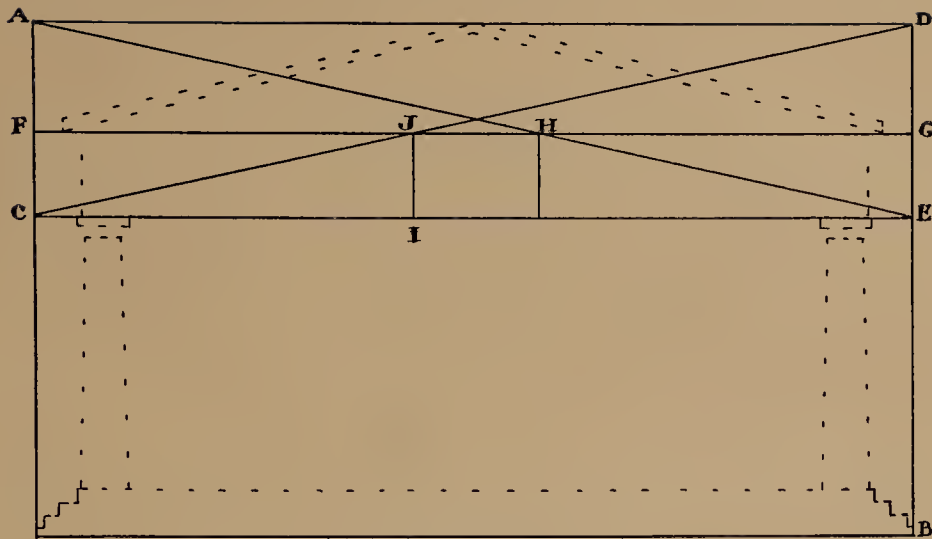


Figure 13.

The façade rectangle, without that of the pediment, AB of figure 14, is composed of two .618 rectangles plus four .236 areas.

AC equals $.618 \times 2$.

CD equals $.236 \times 4$.

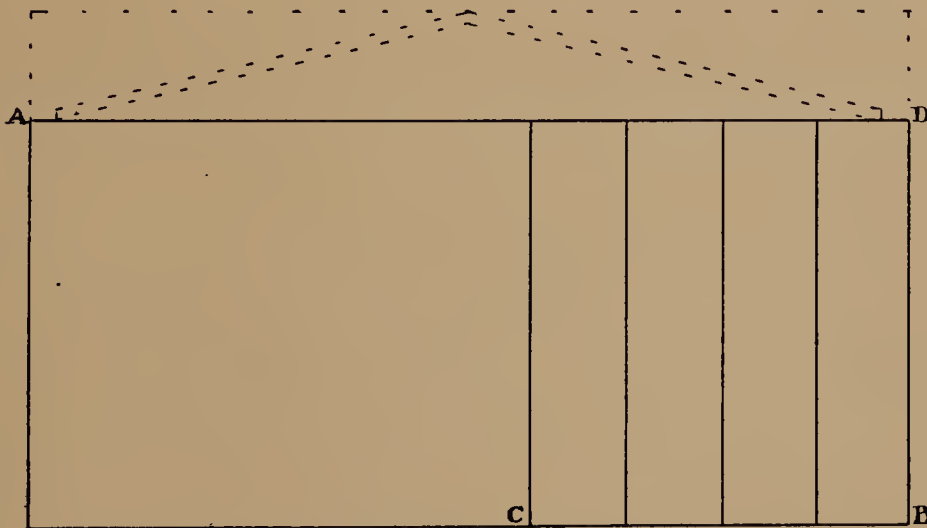


Figure 14.

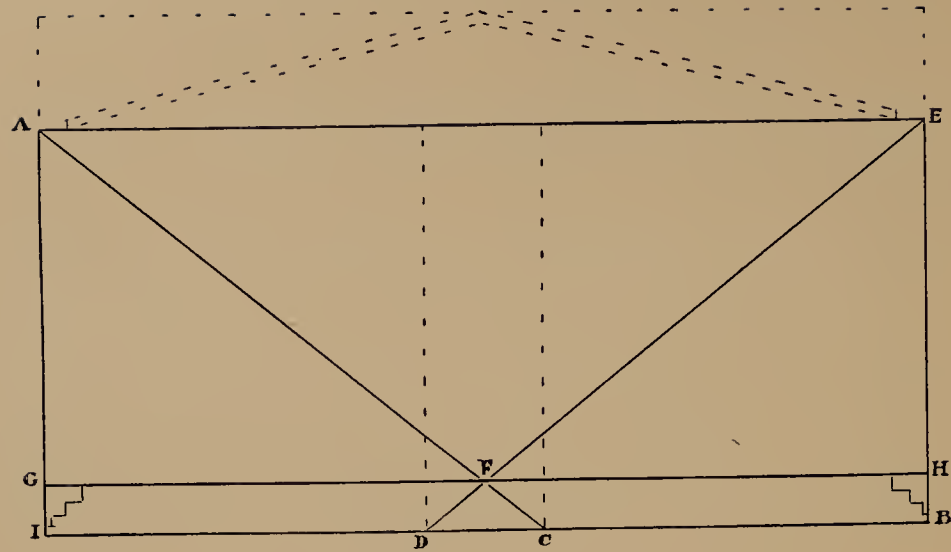


Figure 15.

AE of figure 15 is the top of the entablature while IB is the ground.
 AC is composed of two .618 rectangles while DE is a similar and equal figure which overlaps AC to the extent of CD.

Diagonals to these overlapping areas cross each other at F.

GH, through F, is the top line of the steps.

The area AH is composed of four .618 rectangles.

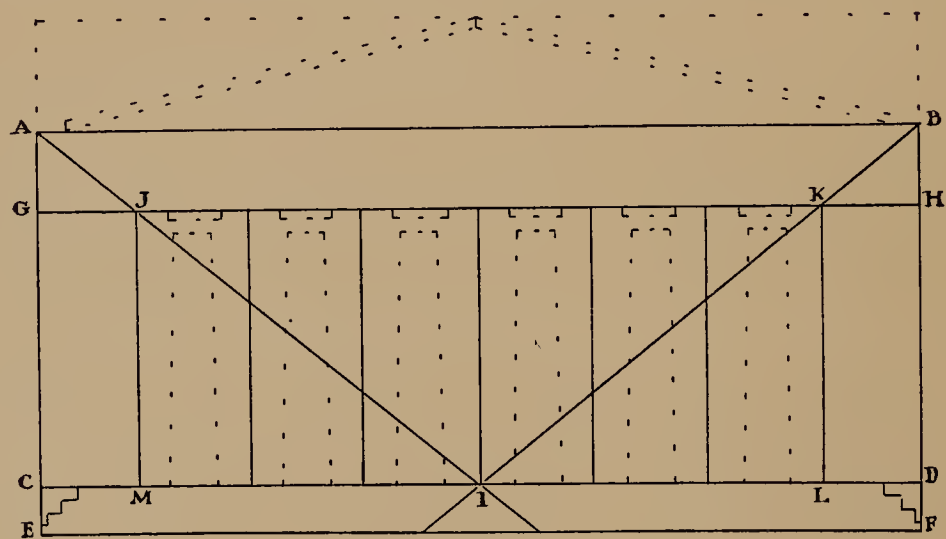


Figure 16.

AB of figure 16 is the top of the entablature.

CD is the top of the steps.

GH is the top of the columns.

AD is composed of four .618 rectangles fixed by the diagonal lines AI, IB.

These lines cut the line at the top of the columns, GH, at J and K.

The rectangle MK is composed of four .618 rectangles. It is also the rectangle which exactly contains the six regular intercolumniations of

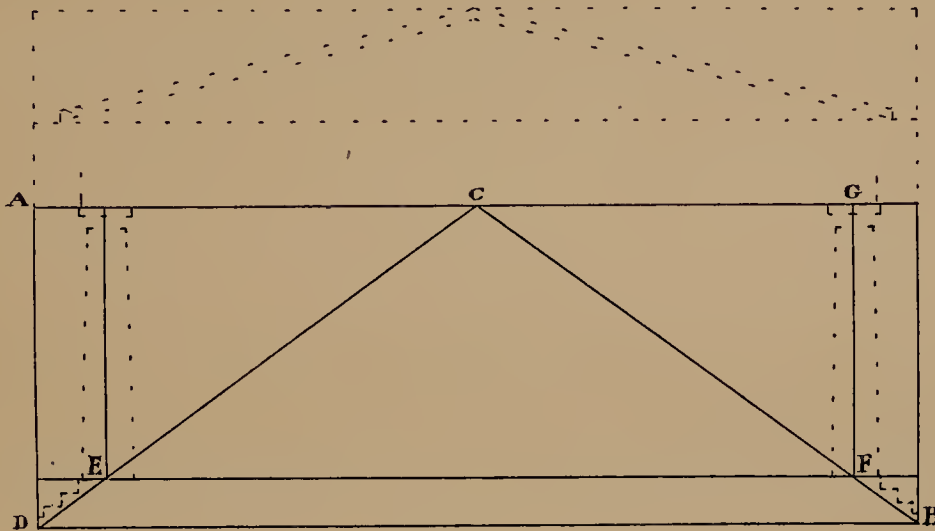


Figure 17.

each façade when a column is placed in the center of each of the six rectangles into which MK is divided.

This rectangle provides the centering of all façade columns except those of the angles. (See figure 16.)

AB of figure 17 is the façade rectangle without the entablature and pediment.

It is composed of four squares and four root-five rectangles. (See figure 9, Chapter One.)

CD and CB are each diagonals to one-half of AB.

These diagonal lines cut the top step line at E and F.

The points E and F are the centers of the angle columns.

Thus the rectangle containing all the column centers of each façade is composed of four squares and four root-five rectangles, because EG is a similar figure to AB.

Six regular column spacings fill the rectangle MK of figure 16.

If this rectangle is divided vertically into six equal parts each will be a

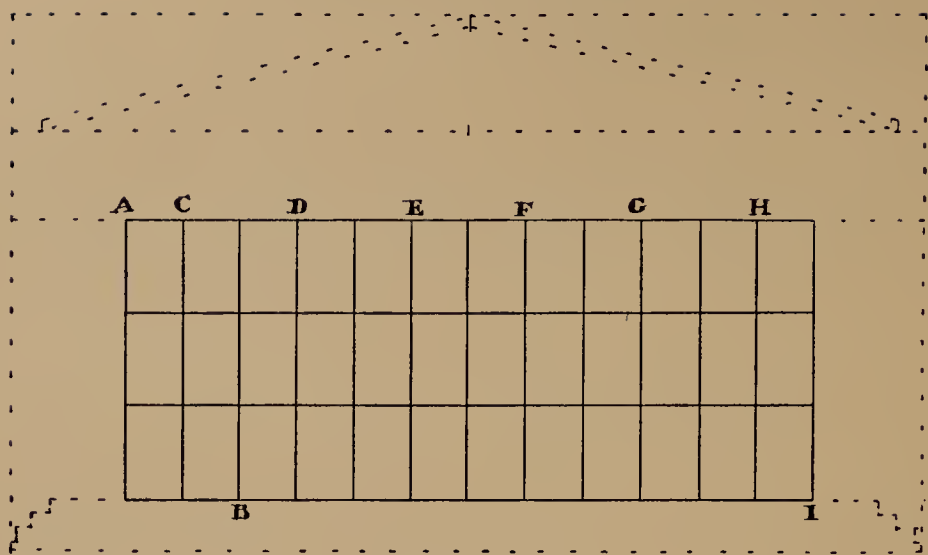


Figure 18.

rectangle composed of two squares and an area similar to one-quarter of the façade rectangle, or;

The entire area AI of figure 18 may be considered as a trellis of .618 rectangles when it is divided horizontally into three equal parts and vertically into twelve equal parts.

A column occupies a space in the center of each one-sixth of AI, as AB. C, D, E, F, G, and H are column centers.

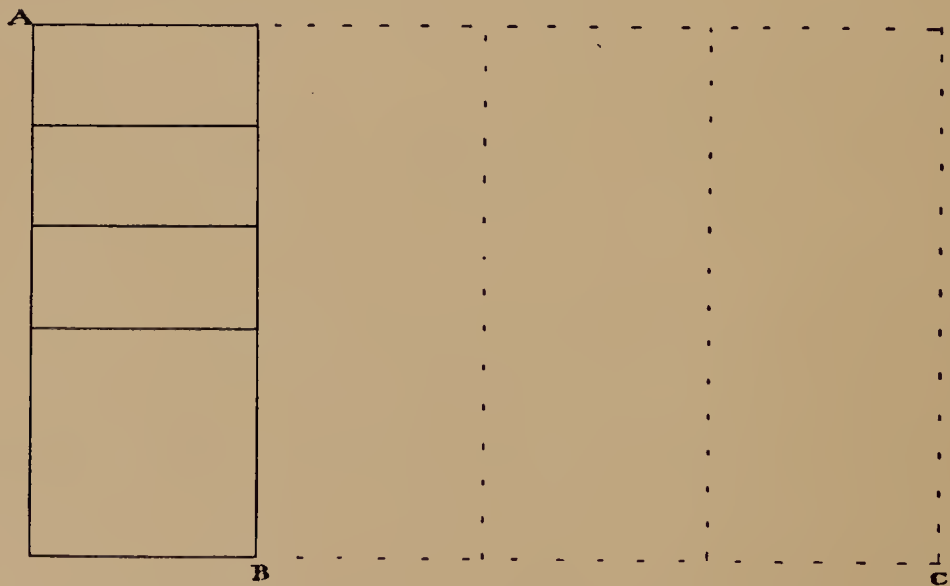


Figure 19.

One-quarter of the façade rectangle (figure 19), as AB of AC, is composed of a square and three root-five rectangles.

The height of the steps with the full width of the building provides a rectangle composed of eighteen squares and two .236 areas.

The rectangle made by the column heights with the full width of the building is composed of three squares and a quarter.



Figure 20.

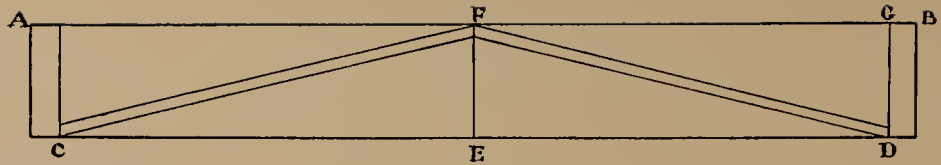


Figure 21.

CHAPTER THREE: THE ENTABLATURE

THE full length of the horizontal cornice of the entablature, including the corona and mouldings, is 104.645 feet. Penrose's measurement is 104.635.

This measurement is that of a straight line joining the two opposite ends of the cornice and leaves out the curve of the pediment floor.

The pattern trellis of proportion seems to indicate that this curvature amounts to about .216 feet. Penrose fixed it at about .184 at the top of the entablature and at about .225 at the stylobate, or top step.

All investigators agree that the plan of the Parthenon must have been in straight lines, the curvature refinements being introduced later.

When a straight line plan is being developed from measurements the curvature is not likely to be noticed until the top of the entablature of a façade is reached, because measurements are fixed by the corners of steps, epistyle and cornice. From the top of the entablature to the peak of the roof, really the top of the sima, the measurement is in the middle of the façade and the curvature increment becomes apparent at once.

This curvature swell in the pediment floor must have concerned the sculptors. It probably compelled them to decrease the height of their figures about two inches, as they might have been developed from a straight line plan.

The height of the pediment from the top of the entablature to the top of the sima is 14.111 feet.* Full width of the temple 111.31 feet. The resulting area, as was explained in the last chapter, is composed of two root-five rectangles plus two similar figures to the façade rectangle.

The actual length of the entablature cornice is shorter than the full temple width by a distance sufficient to make two .236 rectangles, one for either end of the pediment area. (See figure 21; also next page.)

* That is, from a straight line joining the two ends and without the curvature increment.

AB is the full temple width.

EF is the pediment height, including the raking cornice and the sima.

CD is the actual length of the entablature cornice.

AC and BD are .236 rectangles.

The area CG is composed of four squares and two similar figures to the entire façade rectangle; or,

CG is composed of twelve .618 rectangles.

If the sima and raking cornice were included by Phidias in the area within which he composed his pediment groups the proportion trellis upon which he could have worked would not be unlike the arrangement of figure 22.

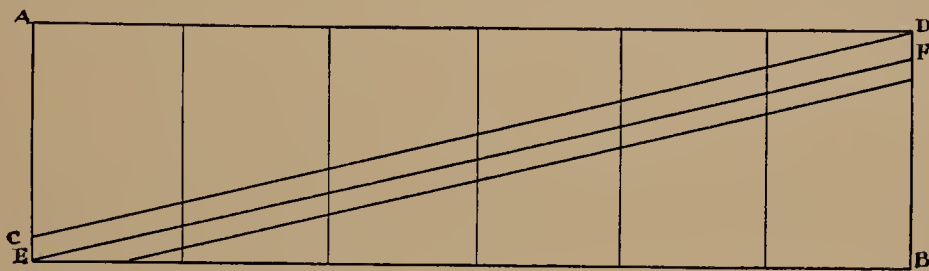


Figure 22.

It will be noticed that neither CD, the top of the sima, nor EF, the top line of the raking cornice, is a diagonal to the area AB; therefore it is more likely that Phidias used the area of which EF is the diagonal.

This line represents the top of the raking cornice and is a diagonal to a rectangle composed of ten figures similar to the column-spacing rectangle.

Thus, the trellis arrangement of the pediment would probably be something like figure 23.

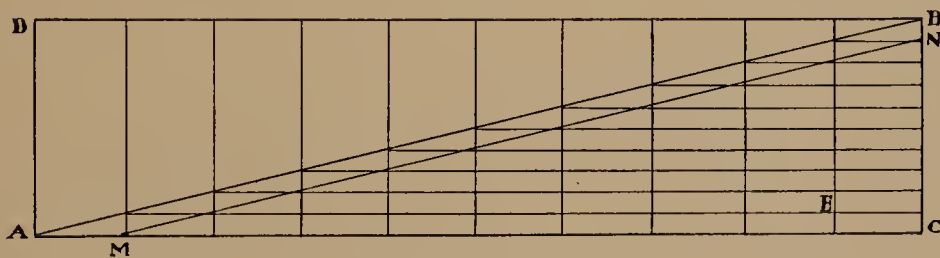


Figure 23.

AB represents the upper edge, while MN shows the lower edge of the raking cornice. It would be immaterial whether the right-angled triangle ACB or MCN were used; they are similar triangles and pattern results would be the same.

CB is the height of the pediment; including the raking cornice but excluding the sima, and curvature.

ACB is one-half of the pediment triangle.

The rectangle DC is composed of ten column-spacing rectangles.

It will be recalled that a column-spacing rectangle is composed of six .618 rectangles. Consequently DC is equal to sixty .618 areas.

Because AB is a diagonal to DC the horizontal lines in ACB divide the area of that triangle into forty-five similar figures to DC, that is, each small rectangle, as CE, is composed of ten similar figures to the column-spacing rectangle.

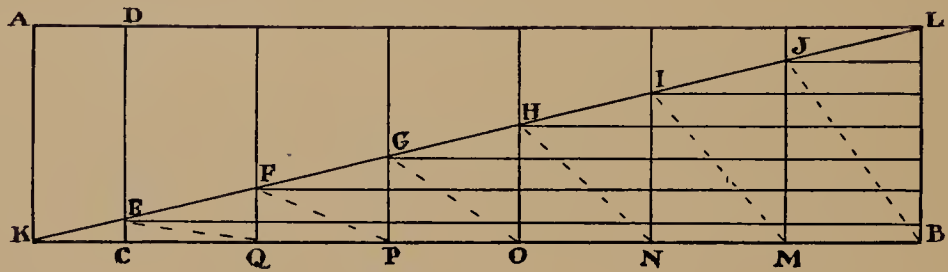


Figure 24.

One-half of the pediment triangle may be arranged as in figure 24.

AC is a similar figure to a column-spacing rectangle.

DB is composed of six .618 rectangles.

The upper edge of the raking cornice cuts the sides of the .618 areas at E, F, G, H, I, and J.

That is, this diagonal line KL fixes similar figures to AB and DB in all of the composing rectangles.

It is difficult to imagine a more thoroughly graphic method of fixing proportion in a design field than that supplied by either figure 23 or 24.

The horizontal and perpendicular divisions may be supplemented by the diagonal lines JB, IM, HN, GO, FP, and EQ, or something like them, thus fixing in the pediment triangle a series of proportional lines of action which we know is characteristic of pedimental composition.

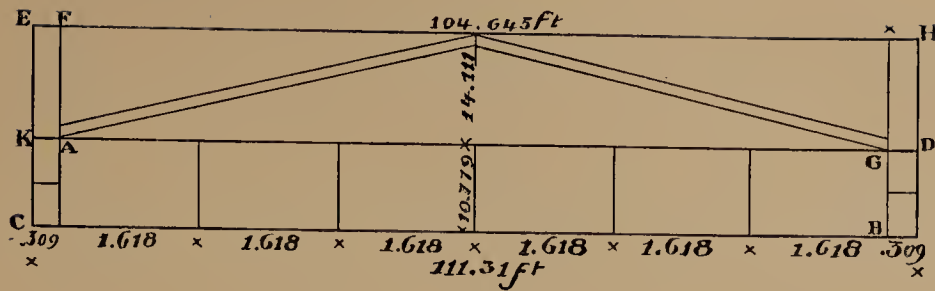


Figure 25.

The proportion trellis for the entablature is fixed by the full width of the cornice as was the scheme for the pediment.

The entablature height divided into the cornice length produces six 1.618 rectangles.

1.618034×6 equals 9.7082.

One column-spacing rectangle has the ratio 2.42705.

2.42705×4 equals 9.7082.

This area is also equal to four column-spacing rectangles.

AC and BD are two small areas on either end of AB which complete the area to the full width of the temple.

Each of these areas is composed of two .618 rectangles.

The area KF is greater than the area AC by one square.

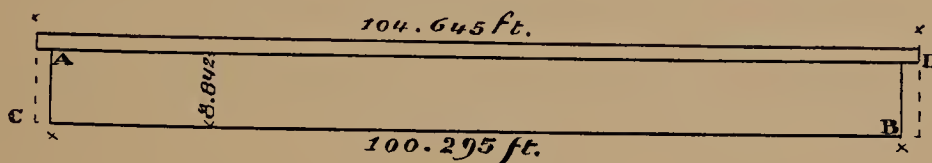


Figure 26.

The height of the frieze and epistyle, that is, the entablature without the cornice, with the full length of the epistyle, provides an area of ten squares and three root-five rectangles. The length of the epistyle is $100.295 \pm$ feet.

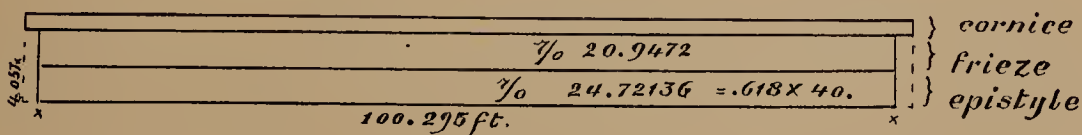


Figure 27.

The first horizontal division of the entablature, after the elimination of the cornice, is at the bottom of the epistyle fillet.

This places the fillet or *tænia* within the area of the frieze, but, as part of the frieze occupies the epistyle territory anyway—*regulæ* and *guttæ* for example—this, in my opinion, is legitimate design practice from the Greek point of view.

The epistyle, full length and height, to the bottom of the *tænia*, provides a rectangle composed of forty .618 rectangles.

The frieze, full length and height, from the bottom of the epistyle fillet to the bottom of the cornice, is an area composed of twenty squares and a figure similar to that of a metope.

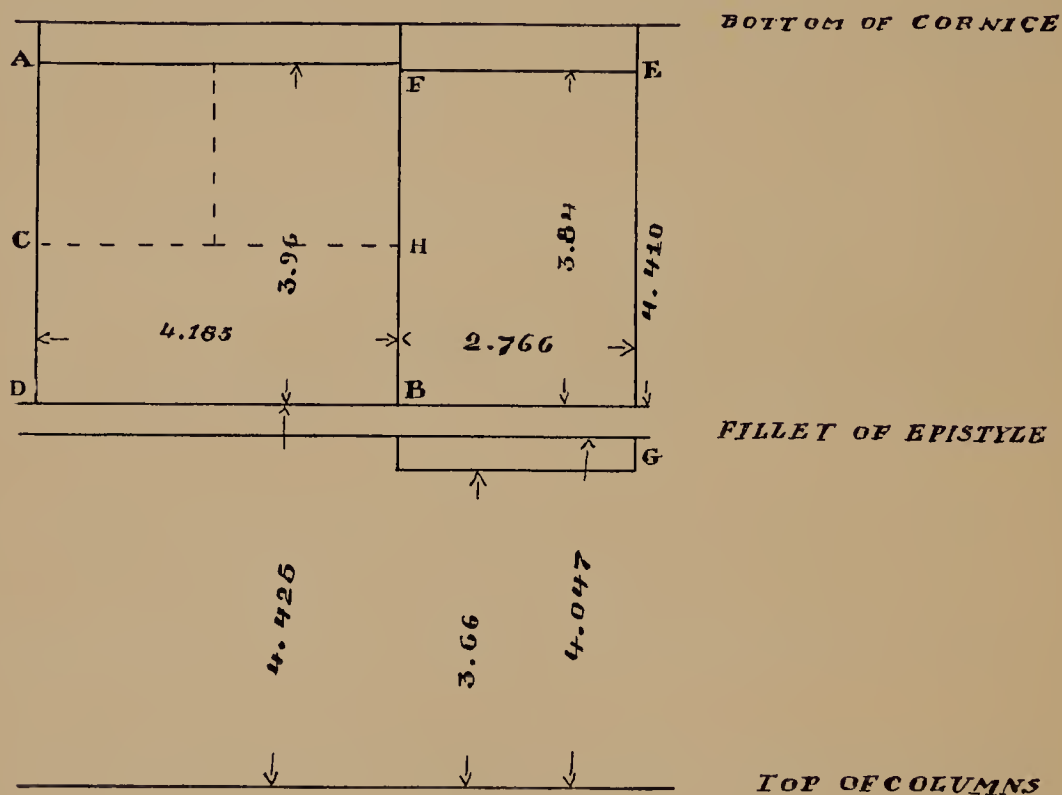


Figure 28.

The area of the metope without its cap, AB of figure 28, is composed of two squares, AH, and a root-five rectangle, CB. The reciprocal of this area belongs to the major frieze area of twenty squares mentioned above.

The areas of metope and triglyph do not equal each other in height so they will be considered separately at first.

There are reasons for believing that the horizontal divisions of the entablature, which we assume to be the most natural, were not the divisions which the Greeks had in mind when they made the plan for the building.

The triglyphs are fairly uniform in width on the façades but the widths of the metopes vary considerably, being much greater toward the center of the frieze than they are toward the ends. Penrose's observation about this I believe is sound—that the increase of width toward the center was made to agree with the natural increase of volume toward the center of the pediment compositions above.

The widths for the fourteen metopes of a façade are: 4.160, 4.066, 4.064, 4.050, 4.282, 4.295, 4.320, 4.375, 4.169, 4.186, 4.195, 4.192, 4.120, 4.121. Total 58.595; mean 4.185.

The mean width of the triglyphs is 2.766 feet—the proportion trellis calls for 2.771. Fifteen equal 41.565 feet.

41.565 plus 58.595, the total for the metopes, equals 100.160 feet for the actual width of the frieze without the contraction which exists above the fillet. This is probably as near as it is possible to get to the intended width as this part of the structure is much affected by the contraction refinement. (See Appendix, Note V, for the real shape of the Parthenon.)

The triglyph was probably intended to be a .618 rectangle, 2.771 by 4.484 feet (FG of figure 28).

Concession may be made to the modern point of view and the entablature divided as the blocks were cut. But I am not convinced that this was the way the original design was made or the proportions fixed. According to this the epistyle is fixed by the top of the fillet. By this division the proportions are:

	<i>Dynamic.</i>	<i>Penrose.</i>
Total height	10.779 feet	10.793 feet
Epistyle	4.432 feet	4.425 feet
Frieze and Cornice	6.347 feet	6.368 feet
		6.347 feet
	Difference	0.021 feet

Total length of cornice 104.645 feet.

Height of epistyle 4.432 feet. Ratio 23.6068, which is $.236 \times 100$, or 2.36×10 .

Total length 104.645 feet.

Frieze and cornice height 6.347. Ratio 16.4808.

This area is equal to 40 column-spacing rectangles. ($.41202 \times 40$.)
The error in this length of over 104 feet is .0025.

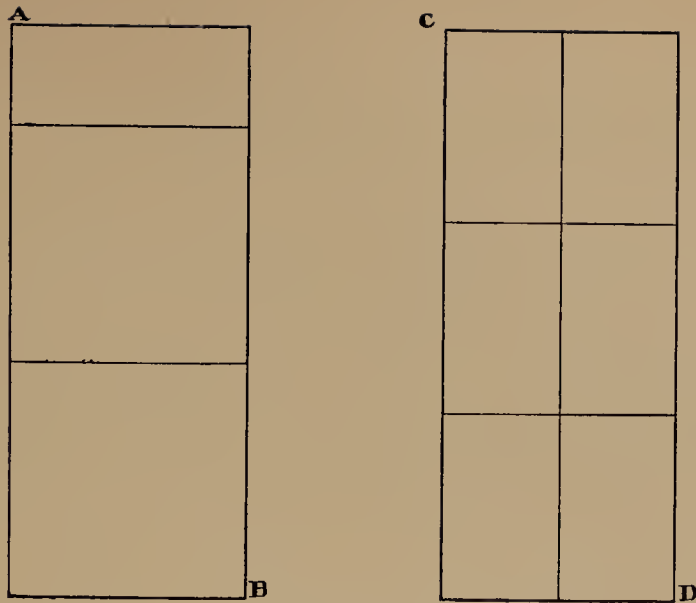


Figure 29.

CHAPTER FOUR: THE PARTHENON COLUMN

THE proportion trellis for the Parthenon column will be understood best if considered in connection with the column-spacing rectangle of the façade.

The connection of this column-spacing rectangle with the proportions of the entire façade has been explained.

AB is the column-spacing rectangle, which may be regarded as composed of two squares and a similar figure to one-quarter of the façade or as six .618 rectangles arranged as in CD.

The proportions of the columns vary slightly according to a definite scheme made to suit the position which the building occupies. The largest columns are those of the eastern angles. The six intermediate columns of the east façade are a trifle smaller.

The angle columns of the west façade are the same as the six intermediate ones of the east end.

Those of the south flank, part of those of the north flank, and the intermediate columns of the west façade are the same. The variations follow closely the unevenness of the ground surrounding the building, the largest being where the temple platform rests immediately on solid rock; the smallest where the ground is lowest.

All of these proportions are variations from a standard which appears to be that of the eastern angle columns.

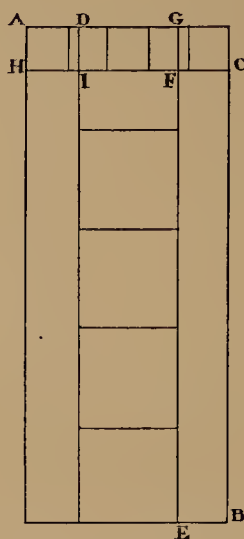


Figure 30.

The development of the standard proportions is illustrated in figure 30. AB is the normal column-spacing rectangle of the façades, 14.11 by 34.247 or 34.246.

AH is 2.822 feet, HC 14.11 feet.

The rectangle AC is composed of five squares.

In the center of AB the distance DG is $6.85 \pm$ feet.

The rectangle DE is composed of five squares.

DE cuts AC at F and I.

DF is a similar figure to AB.



Figure 31.



Figure 33.

Unfinished Capital of the Earlier Parthenon.



Figure 34.

A very early unfinished Doric Capital now lying on the ground within the enclosure of the Theatre of Dionysos at Athens.

This is an illustration of a principle of proportion seldom found in design practice, perhaps nowhere outside of ancient Greece.

If an end and side of any rectangle be divided into an equal number of parts and similar areas be constructed upon these parts, the area of the end and side overlap is a figure similar to the entire rectangle.

DF of figure 30 is similar to a column-spacing rectangle and is the projection of the head of the Parthenon column.

There are six regular column-spacing rectangles on each façade.

AB of figure 31 is the rectangle of the angle column capital and is similar to a column-spacing rectangle.

Divide it vertically into six equal parts.

Each part is composed of four .618 rectangles.

That is, each part is a similar figure to six column-spacing rectangles.

AC is composed of six squares and is a side of the abacus of the capital.

DE is composed of a square and two .236 rectangles, DF, FG, GE; hence DH is composed of two .618 areas.

Draw DH.

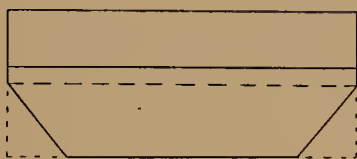


Figure 32.

There are several unfinished Doric capitals on or near the Acropolis at Athens which show that the echinus was developed by a chamfering process. The Parthenon capital at a certain stage of its working must have looked like figure 32, with the exact proportions shown in figure 31.

Figure 33 is a photograph of an unfinished capital belonging to the earlier Parthenon, the uncompleted building which was destroyed by the Persians and now—1920-1921—lying on the ground in front of the Erechtheum.

Figure 34 is an unfinished head in poros stone of an unknown early date and now lying within the enclosure of the Theatre of Dionysos under the Acropolis.

There are several capitals on the Acropolis in a similar condition of finish but of a later date.

These unfinished capitals show that the echinus curve of the Doric

column was worked by straight lines fixed by graphic proportion, these lines representing tangents to a curve, the process of working being a chamfering such as was used when the first faceted column shafts were developed from rectangular blocks; column shafts, for example, like those of the tombs at Beni Hassan in Egypt.

Analyses of hundreds of contour curves of Greek pottery and mouldings point to this process as the only practicable method by which a designer could control exact proportional relationship in curves.

Moreover it is a spontaneous method—a rule of thumb scheme which skilled workmen would discover for themselves. It is a process by which classic Greek curves of all kinds may be drawn either freehand or with mathematical precision.



Figure 35.

AB of figure 35 is one-sixth of the standard capital rectangle; that is, it is composed of four .618 areas.

AC is a square; BD a square and two .236 rectangles.

In the center of DB construct the .618 rectangle EF.

This is done by cutting CB in extreme and mean ratio and making EG equal to the greater segment.

Figure 36 is the .618 rectangle EF of figure 35.

Divide it into four parts horizontally and into six parts vertically.

The major rectangle is now divided into twenty-four rectangles each of which is similar to a column-spacing or angle column capital rectangle.

To the small rectangle EO apply the square EJ.

Draw a line from A to B; also a line from D to C.

The latter line cuts the AB line at K.

Draw a line from D to M and one from F to G. They cut each other L.

Beginning at P draw, freehand, a curve to touch the points Q and R.

This is the silhouette of the curve of the echinus.

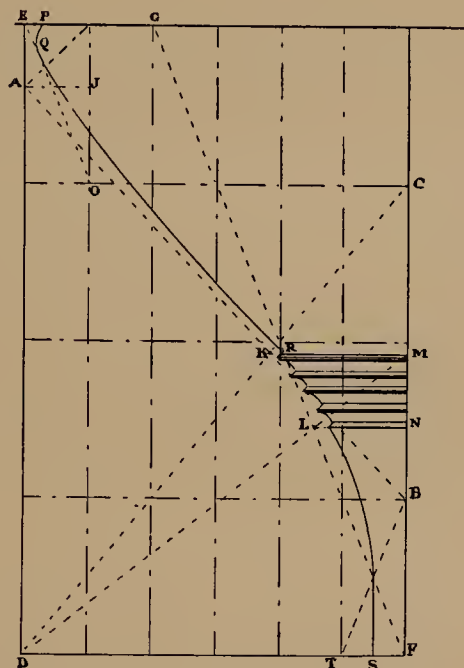


Figure 36.

A skilled draughtsman can draw a curve to touch the points P, Q, and R and obtain practically the same curve every time he tries. There is really but one satisfactory curve which can be drawn to touch these points and the eye will be offended if it is not done correctly. The proof of this is easily made by trial.

Osculating circle arcs may be used to fix the echinus outline or a logarithmic spiral curve cut from celluloid or cardboard, but the freehand curve is the best.

The advantage of having points on tangent lines is that they compel the draughtsman to think ahead of his line. The layman can hardly appreciate the importance of this but it marks the difference between the ancient Greek and the modern artist. The modern artist draws his line by feeling; the ancient Greek by feeling based upon knowledge. That is,

points it is also natural to expect an attempt would have been made to increase or emphasize the weight of the platform where the columns were lighter to make it appear more firmly anchored to the edge of the incline and to give a slight effect of lean toward the top of the hill.

This also was done, as the floor of the building is actually higher at the southwest than it is at the northeast corner (nearly two inches).

The abaci of the two angle columns of the eastern façade are $6.85 \pm$ feet wide.

Those of the intermediate columns of this front have a mean width of $6.75 \pm$ feet while those of the south flank, part of the north flank and the intermediate capitals of the west end have a mean width of $6.606 \pm$ feet.

The two angle columns of the west end have an abacus width of 6.75.

The above measurements are demanded by the proportion trellis; actually the south flank and west end abaci are $6.57 \pm$ but Penrose noticed a peculiarity of the echinus of the south flank capitals which convinced him that the original width of these abaci was 6.606 feet. He supposed "that these capitals were reduced about .036 feet from the breadth they were intended to have. If this quantity were added the measurement would become 6.606."

He also pointed out that an abacus of a north flank capital, near the northeast corner, had too much width for its echinus. This points to an adjustment of the abaci widths, probably after they were in position, to suit the irregularity of the ground.

Penrose found that the echinus curves of all the peristyle capitals were the same.

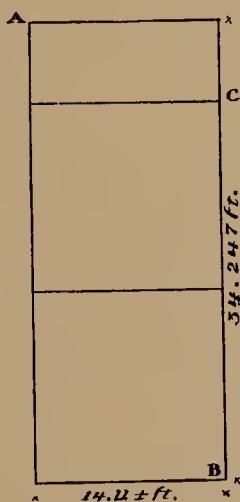


Figure 38.

The rectangle of the smallest heads, $6.606 \pm$ by $2.822 \pm$ is a similar figure to one-quarter of the façade area, that is, it is composed of a square and three root-five rectangles as compared with the angle capital, which is a similar figure to the column-spacing rectangle.

AB of figure 38 is the column-spacing rectangle.

AC is a similar figure to one-quarter of the façade area.

AB is similar to the angle capital.

AC is similar to the smaller capital.

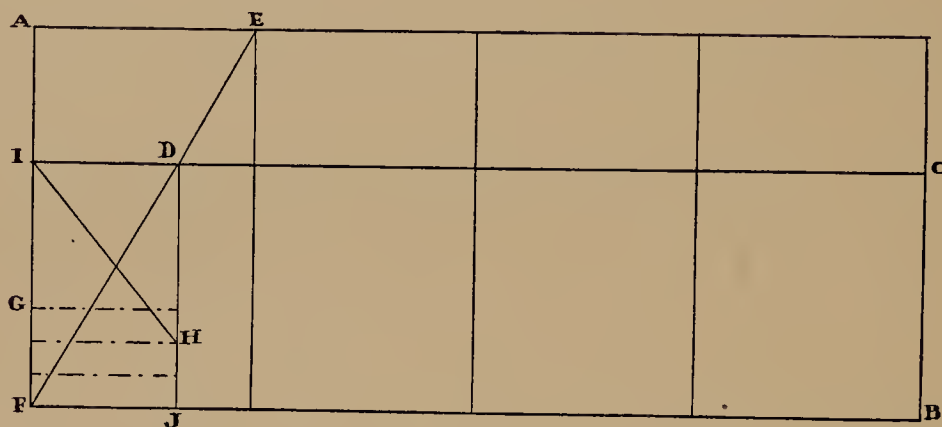


Figure 39.

AB of figure 39 is a similar figure to one-quarter of the façade rectangle and is divided vertically into four parts.

Hence each part is a similar figure to the entire façade area.

IC is the line of the bottom of the abacus.

EF is a diagonal to a similar figure to the façade.

EI, one quarter of AC, is composed of a square and a root-five rectangle.

EF cuts IC at D. Hence DF is a similar figure to the façade area, that is:

DF is composed of the square DG and the three .236 rectangles GJ.

The line HI is a diagonal to two .618 rectangles.

This is the smaller capital rectangle.

AB of figure 40 is fixed by J of figure 39 and is the width of the column shaft, within the flutes, at its juncture with the capital.

BC is a 1.618 rectangle.

CD, fixed by the bottom of the abacus, is composed of four squares.

The entasis curve for a Greek column presents a problem which has not been solved by modern architects. The entasis for a Parthenon column,

or for any column, may be fixed in proportion to the column itself or to the building or to any part of it by a very simple use of the proportion trellis. Unlike most modern absurdities the Greek entasis for the best columns is so subtle that it can scarcely be distinguished in a small drawing, hence the exaggerated column of the diagram.



Figure 40.

AB of figure 41 is the enclosing rectangle of the much shortened column and AC is the capital.

The width of the neck is DH while that of the base is EJ.

Draw a line from I to E and one from D to F.

These two lines cross each other at G.

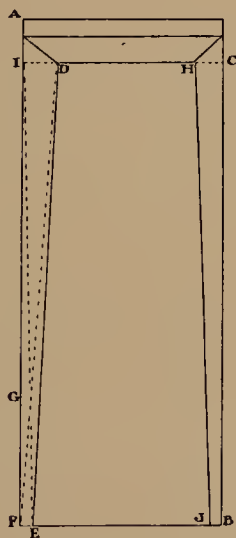


Figure 41.

DG and GE will be quasi tangents which will be guides for the draughtsman while drawing a freehand curve from D to E which will gradually fall away from DG until it reaches a point opposite G; it will then gradually approach GE until it unites with it at E.

This is an explanation of the much discussed "conic" curves of the Parthenon which enthusiasts have tried to prove were used in this building before they were discovered.

The curve may be made to reach its greatest swell at any point along DE by varying the proportions along FA, or ID for the lines IE and DF.

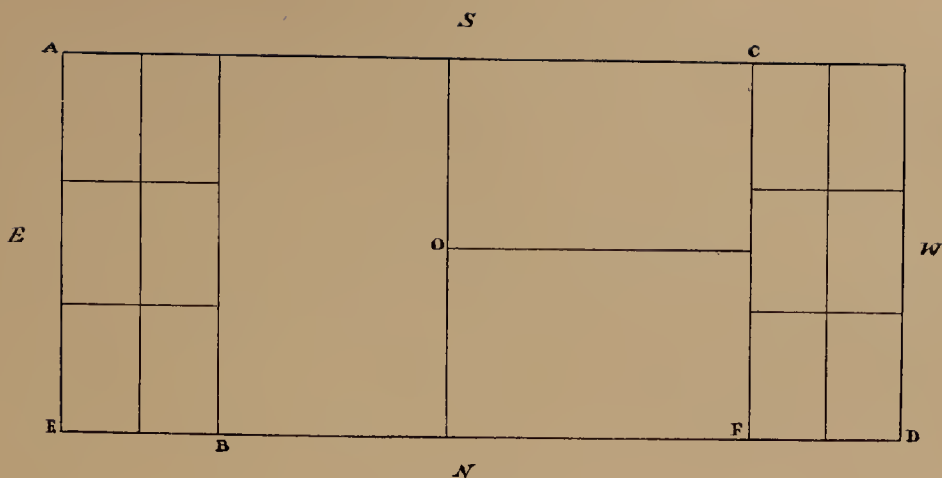


Figure 42.

CHAPTER FIVE: THE TEMPLE FLOOR

THE rectangle made by the top step of the Parthenon may be said to represent the floor of the temple. It is composed of .618 rectangles arranged like those of the regular column-spacing rectangle.

AB and CD are similar figures to the regular column-spacing rectangle.

BC is similar to a column-spacing rectangle minus a square.

The ratio value of AB and CD is 2.427.

The ratio value of BC is 1.427.

The east top step measures 101.341, the west 101.361 feet.

The north top step measures 228.154, the south 228.141 feet.

It has long been assumed that the relation of the end to side top step of the temple was four to nine, that is one to two and a quarter.

This is not true. The amount of error is too great.

The real relationship is 1 to 2.25109 and the real rectangle measures 101.347 by 228.141.

An interesting feature of the proportion trellis is the light it throws on certain well-known irregularities connected with the cella. These are:

The difference in depth between the east and the west porches.

Difference in tread between the east and west bottom steps.

Difference in tread between the north and south steps.

The proportion trellis suggests that these variations were intentional and were probably made after the building was well under way.

The depth of the east porch was increased by moving the east cella wall

about four inches to the west and by narrowing the east step tread about .13 feet. Also the east porch columns were made considerably smaller than those of the west porch.

Some of these changes appear to be connected with the statue of Athena, as its dead center was shifted to the west about the same amount as that of the east wall shift.

Possibly Phidias decided that he needed more porch space and more light or space for his statue.

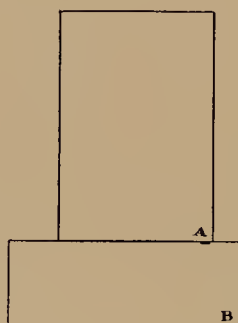


Figure 43.

When the Greeks fixed the different steps of the temple platform, the centers of the columns, and the walls of the cella they were careful to define their exact positions by setting lines chiselled on the previous course of masonry. In the Parthenon these lines were buried about a centimeter under the edges of the superimposed stones and are now visible wherever the walls or steps have been disturbed.

The diagram shows a section of a wall foundation block and of an orthostate or stone of the first wall course.

The setting lines buried under the edge of the stones are marked A and B.

The setting line B is interesting because it shows the proportion of the cella interior before the foundation blocks for the walls were put in position, that is the rectangle KH of figure 44.

The rectangle AE of figure 44 is the same as that of BC of figure 42. It is a 1.427 area.

AD is the full width of the temple floor, 101.347 feet.

DE is 144.627 feet.

BF and GH are squares.

IF and MH produced to K and L fix the square KM.

IJ is a square and JF two .618 rectangles.

KH is the interior of the cella as fixed by the first setting lines, that is, those for the foundation blocks for the walls.

The rectangle KH brings out clearly the greatest irregularity in the building.

The line KL lies immediately under the west side of the east cella wall;

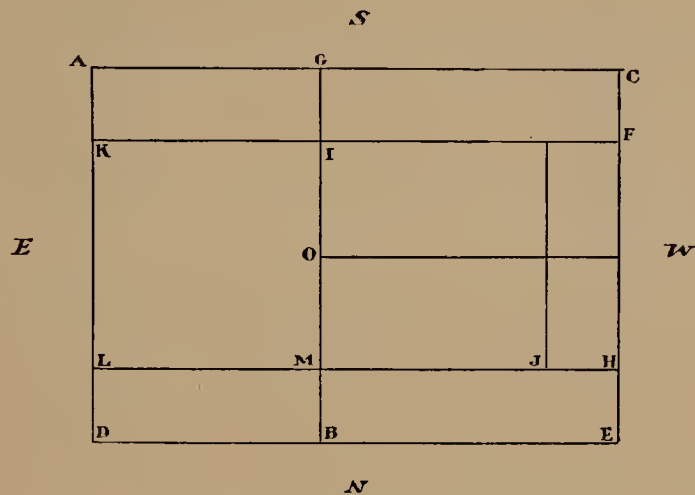


Figure 44.

on the other three sides, KF, FH, and HL are some distance inside the wall lines.

The east wall should be the same distance away from KL that the west wall is away from FH. That is, the east wall is about four inches closer to the west wall than it should be.

This irregularity exactly accounts for the greater depth of the east cella porch.

It thus appears that the original plan called for a cella interior of exactly the same proportion as the rectangle KH. Instead of this rectangle, composed of the two squares KM and IJ and the two .618 rectangles, JF, the floor of the interior, as it now is, is composed of a square and seven root-five rectangles arranged as in figure 62, Chapter Eight.

At the point O there is a rectangular hole the center of which is supposed to be the dead center of the statue of Athena. As a matter of fact the dead center is about four inches to the west of O if we regard KH of figure 44 as the original plan of the cella interior. That is, the shift of the east

wall affected the entire column-centering rectangle of the interior of the Naos, including the dead center of the statue.

The lower step of the cella makes a rectangle inside the top step of the temple.

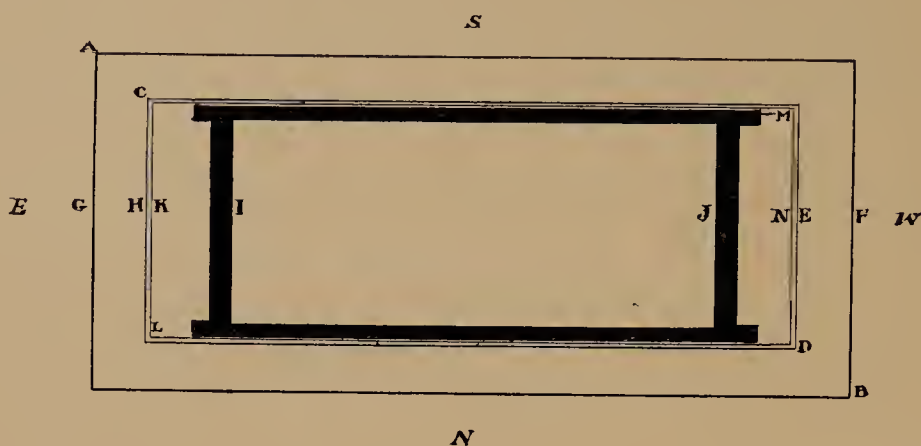


Figure 45.

AB of figure 45 is the temple top step.

CD is the rectangle of the bottom step of the cella.

CD is nearly if not exactly in the center of AB.

The distances GH and FE are almost exactly the same, $15.88 \pm$ and $15.87 \pm$ feet.

Inside of CD there is the rectangle of the top step of the cella. This rectangle is not truly centered with AB and CD.

The heavy lines are the walls of the cella.

LM is the rectangle of the cella top step.

The distance from K to I is 17.78.

From N to J it is 17.33 feet.

Difference 00.45 feet.

The line AB of figure 46 coincides exactly with the inner side of the east cella wall.

CD does not coincide with the inner side of the west cella wall.

The difference is the difference between the depths of the east and west porches, namely, .45 feet, more or less.

CD is the setting line of the wall foundation blocks.

The foundation blocks for the east wall are entirely irregular with the

north, south, and west blocks. For these other three sides they are normal and follow a definite plan.

These irregularities in no way interfere with the dynamic proportions of the building as it now stands. But the point is interesting because it throws light on Greek design practice.

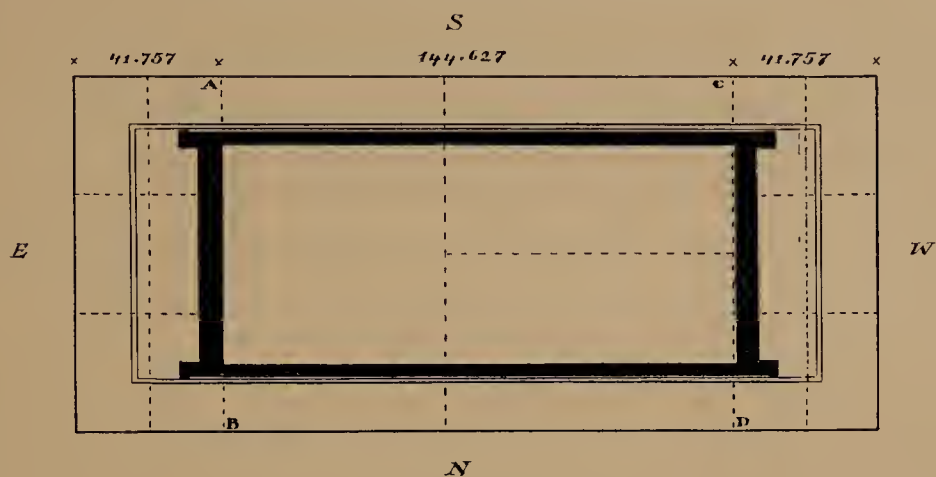


Figure 46.

Penrose thought that the east porch was finished first and proved to be unsatisfactory, the west porch being then changed to suit the judgment of the builders.

The reverse of this is more probable from my point of view; at any rate the east porch was made deeper by moving back the cella wall .45 feet and decreasing the tread of the east cella step from .139 to .126 feet.

The difference between the widths of the cella step treads, east and west and north and south, has been explained by Professor Hill of the American School at Athens, as due to the use of second-hand material from the uncompleted Parthenon destroyed by the Persians. He has identified several blocks of marble belonging to the first Parthenon in the cella steps of the present building and believes that the departures from the original plan were made solely to utilize this old material. I regard this hypothesis as improbable for the reason that I cannot imagine the Greeks changing the plan of such an important building as the Parthenon merely to use a few old blocks of marble when they had a mountain of this material so close at hand.

It seems to me more likely that the cella shift to the north was made to counteract the effect of the steep incline at the south, as columns were made lighter for the same reason. The narrower step tread at the east end of the cella, I believe, was connected with the difference between the east and west porches of the cella. These departures from the original plan, made for the reasons I have given, enabled the builders to use some of the stones from the earlier building. In other words the plan was not changed to accommodate old stones—they were used only when a place for them existed. (See Note VI on the Greek foot.)

CHAPTER SIX: THE TEMPLE FLOOR AND ELEVATION

MUCH of the interior elevation of the Parthenon and all of the roof have been destroyed, hence a great deal of the most interesting part of the building is beyond proportional analysis of any kind.

However, there is enough left at the west end of the cella to fix the height of the ceiling with considerable exactness.

The ceiling is fixed by the under side of the rafters as it usually is in other buildings. This excludes the coffers.

A cross section of the temple interior, from the edge of the top step or stylobate to the ceiling, provides a rectangle with which we are familiar because of its appearance in other proportions. This is the figure which is similar to one-quarter of the façade rectangle, namely, the area composed of a square and three root-five rectangles.

The width of the temple floor is 101.347.

The height from the floor to the ceiling is 43.28 feet.

The three dimensional figure which the temple floor and ceiling represent would be a box. This is what the temple might be said to be if the roof, pediment, cornice, and platform of steps were removed. Thus a Greek Doric temple appears not unlike a sarcophagus, such as that of Alexander the Great or a late Lycian tomb.

The cella, with its subdivisions, would look like compartments within the box.

There are four columns on each front of the cella and a cross section at either front is shown in figure 47.

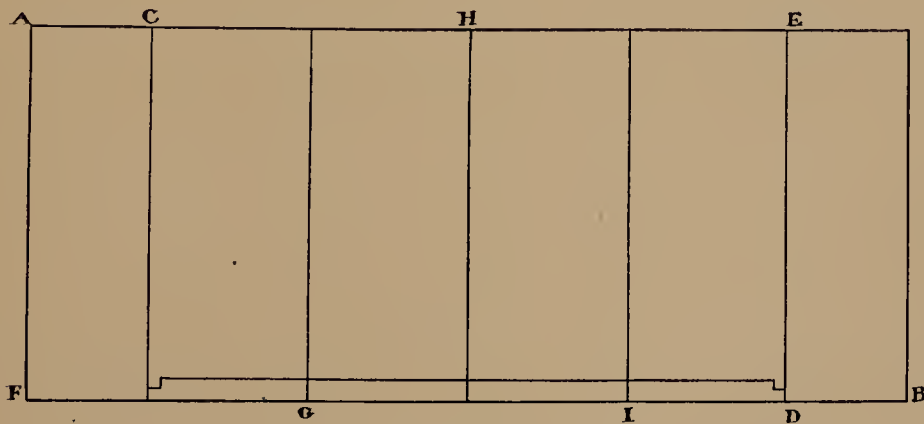


Figure 47.

AB is similar to one-quarter of the façade rectangle.

If the width of the bottom step of the cella be used to fix the rectangle CD, it is composed of four rectangles similar to that surrounding the statue in the Naos, that is, four 2.36 rectangles or forty .236 areas.

The areas CF and EB are each composed of five .618 rectangles.

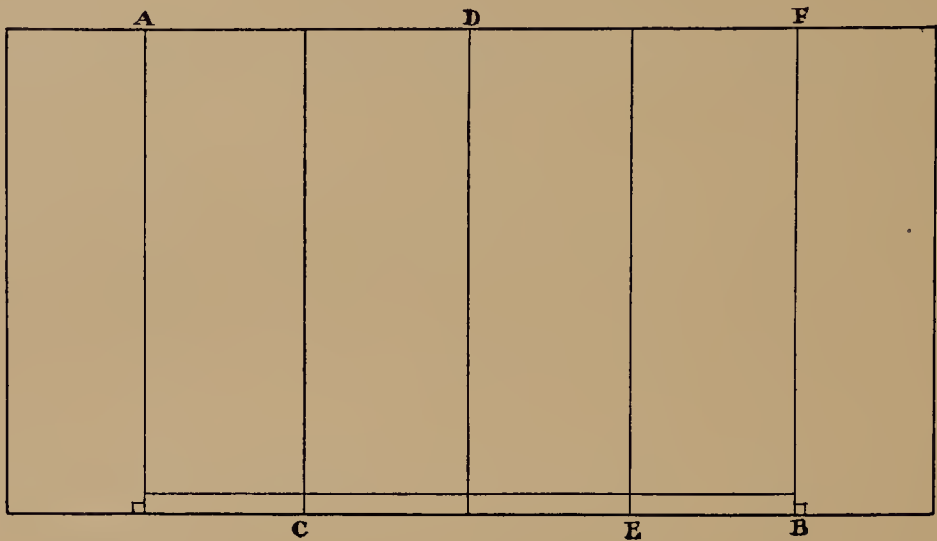


Figure 48.

If the width of the top step of the cella be used to subdivide the area of the cross section the result is the rectangle AB of figure 48. This is composed of four rectangles similar to those of the column-spacing areas of the façades.

If the line of the top step of the cella, that is, the cella floor, be produced



Figure 49.

to A and B, as in figure 49, the outer lines of the stylobate, the area of the cross section CD is divided to fix the rectangle CB.

This rectangle is composed of four .618 areas or is a similar figure to the group of six column-spacing rectangles of the façades. $.618 \times 4$ equals $.412 \times 6$.

The area EF is composed of four root-five rectangles.

Each of the areas AE and BG has the reciprocal value of .34164, that is, $.17082 \times 2$.

The reciprocal .34164 represents the ratio 2.927. This represents two squares and a .618 rectangle plus its half, $.618 + .309$.

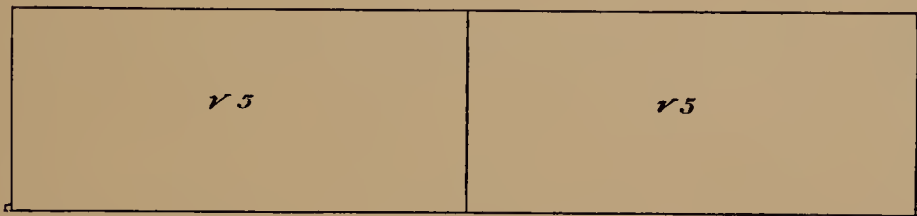


Figure 50.

THE CELLA PLAN AND ELEVATIONS

EITHER the curvature of the floor and ceiling of the temple or the irregularity of the steps of the cella affects elevation measurements of the latter slightly.

The height of the cross section as fixed by the proportion trellis is 43.28. The cella height for its flanks seems to be about two-tenths of an inch greater.

The width of the cella upper step is 71.36. Penrose made it 71.33, but he failed to notice sufficiently that the tread of the lower south step is considerably wider than that of the north.

The entire cella floor, indeed the whole volume of the cella, must be a trifle off center; that is, it is shifted a little to the north and toward the solid rock of the Acropolis. This may be a refinement introduced by the architects for much the same purpose as was the reduction of the column capitals of the south flank.

The length of the cella top step of the flank is 193.733 feet. This measurement with that of the temple floor-to-ceiling height makes a rectangle

composed of two root-five areas, or, it is a similar figure to the pediment and entablature projection.

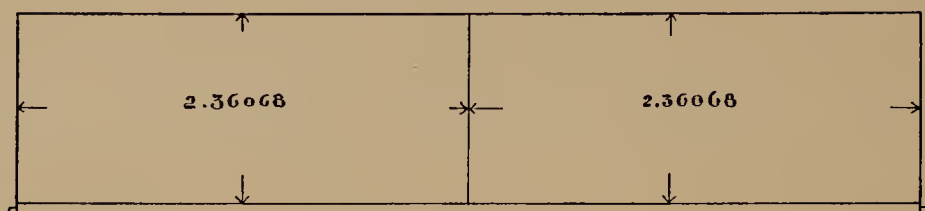


Figure 51.

The same length, 193.733 feet, with the distance from the cella floor to the ceiling provides a rectangle composed of two similar figures to the column-centering rectangle of the Naos, namely, two 2.36 rectangles as in figure 51, or twenty .236 areas.

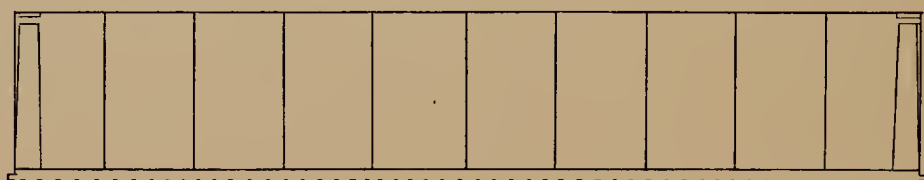


Figure 52.

The height of the cella columns with the flank length of 193.733 feet provides a rectangle composed of ten similar figures to the façade rectangle, as in figure 52.

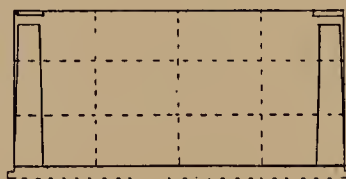


Figure 53.

The height of the cella columns with the width of the top step makes an area composed of .618 rectangles arranged as in figure 53.

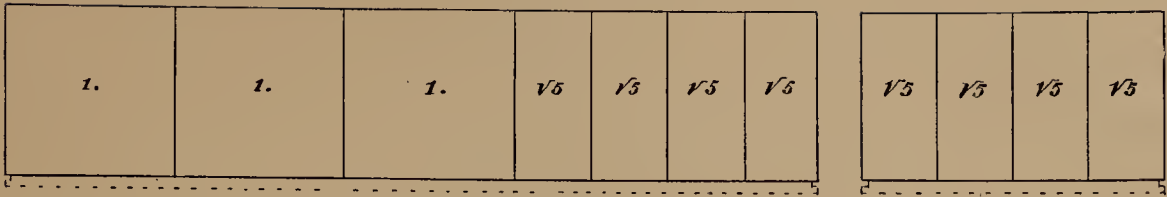


Figure 54.

The flank length of the bottom step of the cella is 196.383 feet. If this measurement is used with that of the cella floor-to-ceiling height the result is a rectangle composed of three squares and four root-five rectangles.

If the full width of the bottom step and the cella floor-to-ceiling height are used for another rectangle it is composed of four root-five rectangles.

Hence the flank rectangle is three squares longer than that of the front.

CHAPTER SEVEN: THE PANATHENAIC FRIEZE AND THE CELLA

THE most interesting part of the exterior of the cella is the celebrated Panathenaic frieze.

If we could recover the actual plan by which this masterpiece was laid out, proportioned, and kept in harmony with the rest of the temple it would be thrilling. But no matter what the plan may have been no one can say how an artist's taste and judgment would be influenced by a proportion scheme.

However, the personal equation is outside the scope of this book. Interest now lies in the organized background upon which taste, judgment, and design ability displayed itself.

The total length of the frieze is 524 feet 1 inch.

The height as measured by Penrose *in situ* is 3.324 feet.

"The length of each end of the frieze was 69 feet 6 inches (21.183 meters). The length of each long side was 192 feet 6 and $\frac{1}{2}$ inches (58.685 meters). Entire length 524 feet 1 inch (159.74 meters)." *The Sculptures of the Parthenon*, by A. H. Smith, page 51.

Smith makes the height of the frieze "nearly three feet, four inches." That is, nearly forty inches.

The proportion trellis calls for 39.911 inches, or 1.0137+ meters.

Penrose measured the height *in situ* and found it 3.324 feet. The proportion trellis demands 3.326+ feet. The difference is about half a millimeter, which is negligible.

The scheme for the entire frieze is a very simple arrangement of square and root-five rectangles.

For each long side there are exactly forty squares and forty root-five rectangles.

For each short end there are eighteen squares and two square and root-five rectangles.

ARITHMETICAL PROOF

LENGTH of each long side of the frieze, 58.685 meters.

Height, 1.01376±.

$\frac{58.6850}{1.01376}$ equals 57.8884+.

Square and root-five rectangle equals 1.4472.

1.4472×40 equals 57.8884+, equals the ratio for each long side.



Figure 55.

Frieze of the Parthenon, procession of horsemen. Acropolis Museum, Athens.

Courtesy of the English Photographic Company.

Length of each end of the frieze, 21.183 meters.

Height, 1.01376±.

$\frac{21.183}{1.01376}$ equals 20.8944, equals 18 squares plus 1.4472×2 .

We may also call this 20 squares plus two root-five rectangles.

20.8944×1.01376 (the height of the frieze) equals 21.18190+. Error 1 centimeter and 1 millimeter.

This slight error is accounted for by the angle turn at the corners of the frieze.

These areas, of course, may be arranged in many ways to suit composition requirements, but these are the outstanding proportions of the ground on which the sculptors worked.



Figure 56.

From the ratios of the front and flank elevations of the top and bottom steps of the cella it is a simple matter to calculate the ratios for the ground plans.

The ratio for the width of the top step and temple floor-to-ceiling height is 1.64808.

For the flank, it is 4.472.

The former divided into the latter results in 2.7135; that is, two squares and one-half of 1.427.

The area of the fraction is composed of .618 rectangles arranged as in figure 56. (See the temple floor pattern, of Chapter One, figure 4, page 8.)

A bottom step front elevation rectangle is composed of four root-five rectangles. (1.788854.)

A bottom step flank elevation is composed of three squares and four root-five rectangles. (4.788854.)

The lesser into the greater ratio results in 2.677 for the bottom step rectangle. This represents an arrangement of root-five rectangles and a square as in figure 57.

AB is composed of a square and four root-five rectangles.

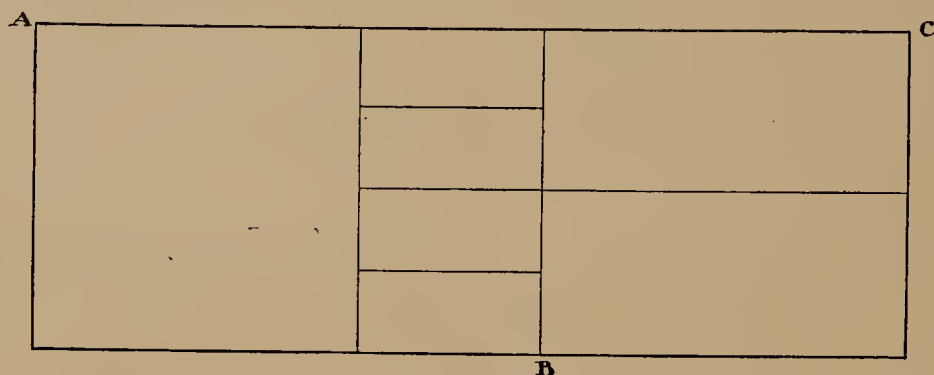


Figure 57.

BC is composed of two root-five rectangles.

AB is an interesting figure because it is the proportion of the floor of the Naos.

The proportion for the entire floor of the interior of the cella is much like figure 57. The only difference is that of BC, which is composed of three root-five rectangles instead of two.

As for the area of the rectangle of the upper steps these ratios suggest that the true width of the bottom step was 73.36 instead of Penrose's 73.4.

The lengths 193.733 and 196.383 were fixed by Penrose for the upper and lower steps respectively. These I have adopted but they may not have been the lengths intended as is suggested by the discrepancy between the treads of the east and west steps, 1.26 for the former and 1.39 for the latter.

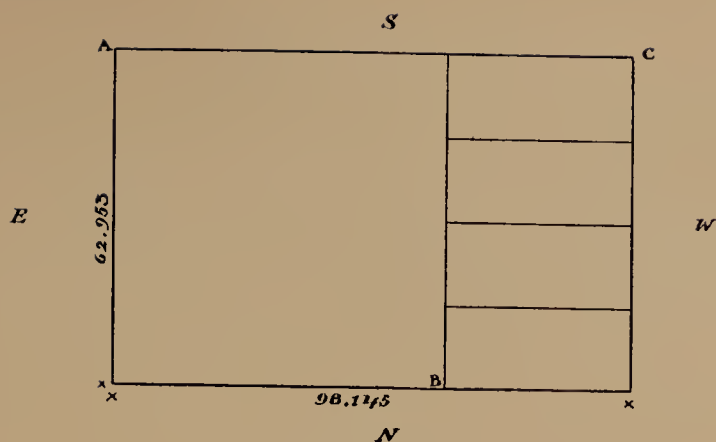


Figure 58.

CHAPTER EIGHT: THE NAOS

THE Naos is probably the most picturesquely interesting part of the temple because it was the room which contained the Phidian statue of Athena.

Everything above the floor of this room at the east end has disappeared except a few of the first wall course stones. The only data available now for an examination of its proportions is that furnished by the poros stone foundation for the statue pedestal or platform, the foundation blocks for the columns which formed a rectangular enclosure for the statue, a slightly sunken rectangle within that formed by the column centers, a rectangular hole in the center of the poros stone foundation and most of the floor blocks outside of those on which the columns stood.

As it is today the floor of this sacred chamber might be likened to an enormous drawing table of smoothed Pentelic marble on which the plan of the room exists as it was fixed by the builders.

From the inner surface of the east to that of the west partition wall the distance is 98.145 feet.

From the inner surface of the north to that of the south wall it is 62.953 feet.

Division of the lesser into the greater of these measurements shows that the rectangle made by the entire floor is composed of a square and four root-five rectangles.

AB of figure 58 is the square, whose sides are equal to the width of the room.

BC is composed of four root-five rectangles.

Within the rectangle of the room, with one end abutting the inner side of the east wall, is the rectangle formed by the centers of the columns which surrounded the statue.

This rectangle is 36.22 feet wide, from north to south, and 85.5012 feet long from east to west (figure 59).

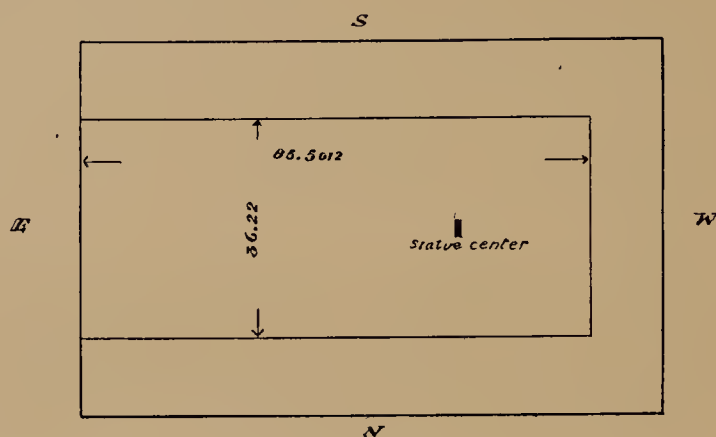


Figure 59.

There were ten columns on each long side, counting the southwest and northwest angle columns, and three on the west end, twenty-three in all.

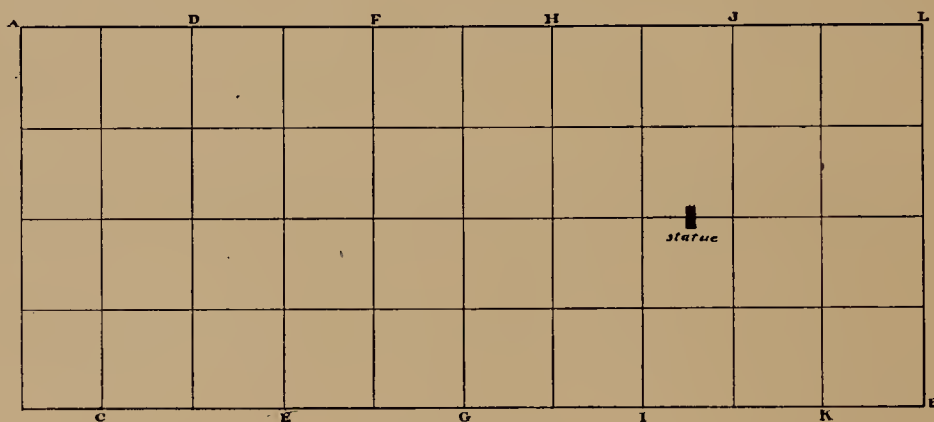


Figure 60.

If these column centers are used as points from which to draw lines across the intervening space from north to south and from east to west the resulting web is a dynamic proportion trellis so simple and so accurate that any other interpretation of the plan is impossible (figure 60).

If the designers of the Parthenon had taken pains to explain their

graphic proportioning method it could not have been simpler or clearer than the diagram fixed by these lines joining the column centers.

It will be noticed that there are ten subdivisions of the length of the rectangle.

Eight of these are equal. The strip AC on the east is narrower than the others while the strip KL on the west is wider.

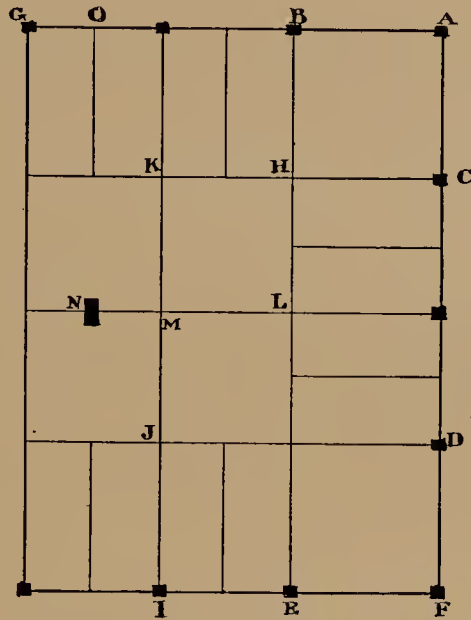


Figure 61.

But AC and KL added exactly equal two of the regular strips.

Each of the eight regular strips is a .236 rectangle, while the two odd strips, together, equal a .472 area or $.236 \times 2$.

The angle columns at the southwest and northwest corners have a greater separation from the next adjoining columns than have the regularly spaced columns, enough to make the distance CD of figure 61 equal GB, that is to make the area DH equal and similar to GH.

BC and ED are squares.

GI and IB are each .236 rectangles.

.236 is the reciprocal of 4.236, that is, two squares and a root-five rectangle.

The lines from the column centers of the western end automatically define the two squares, HJ.

Hence the areas KB, EJ are composed of two root-five rectangles, or, both areas added equal one root-five figure.

Also CL, LD are each composed of two root-five areas.

And the area AE is composed of two squares, AH, ED, and four root-five rectangles, HD.

AI is composed of two squares, AM, MF.

N is the centering hole of the statue base in the center of the .236 rectangle GI.

IO is half of IG, hence the statue center is on the side of a .618 rectangle, OF.

.236 divided by 2 equals .118.

.118, OI, plus two squares, IA, equals .618.

The rectangle made by the entire floor of the interior of the cella, which includes the floor of the Naos and that of the Maiden's room to the west, the *Parthenon*, is composed of the square and four root-five rectangles of the Naos, plus three root-five rectangles. (The Parthenon in Greek means the Maiden's room.) (See figure 62.)

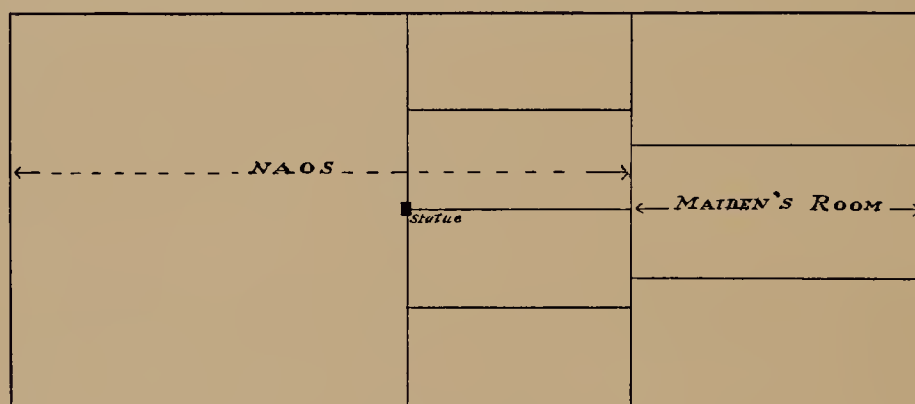


Figure 62.

There were four columns grouped near the center of the Maiden's room, but there are no remains of them whatever. Judging by the size of the foundation blocks, which are still in place, they must have been of considerable height and it has been suggested that they were Ionic.

One of these blocks has a smaller one-way measurement than the others; consequently the column on this block could not have been exactly centered. From the other three the exact centering is fixed, however. The

centers of these columns stood on the four corners of the rectangle which has appeared so frequently in the proportions of the temple, namely, that composed of .618 rectangles arranged as in the diagram of figure 63.

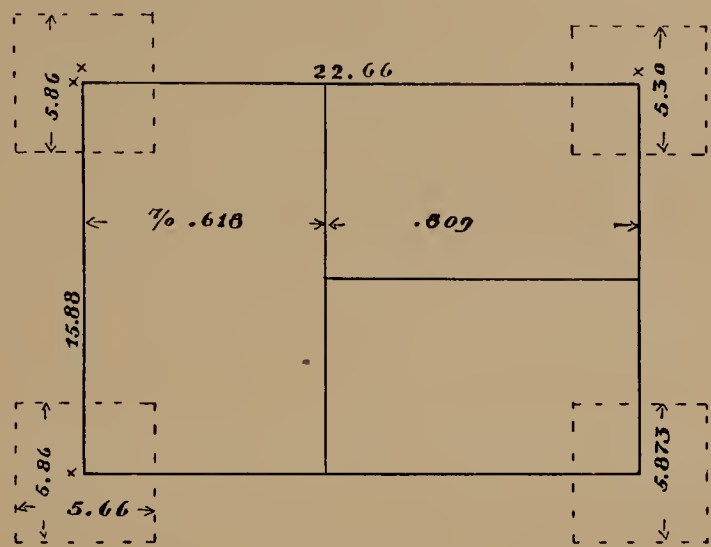


Figure 63.

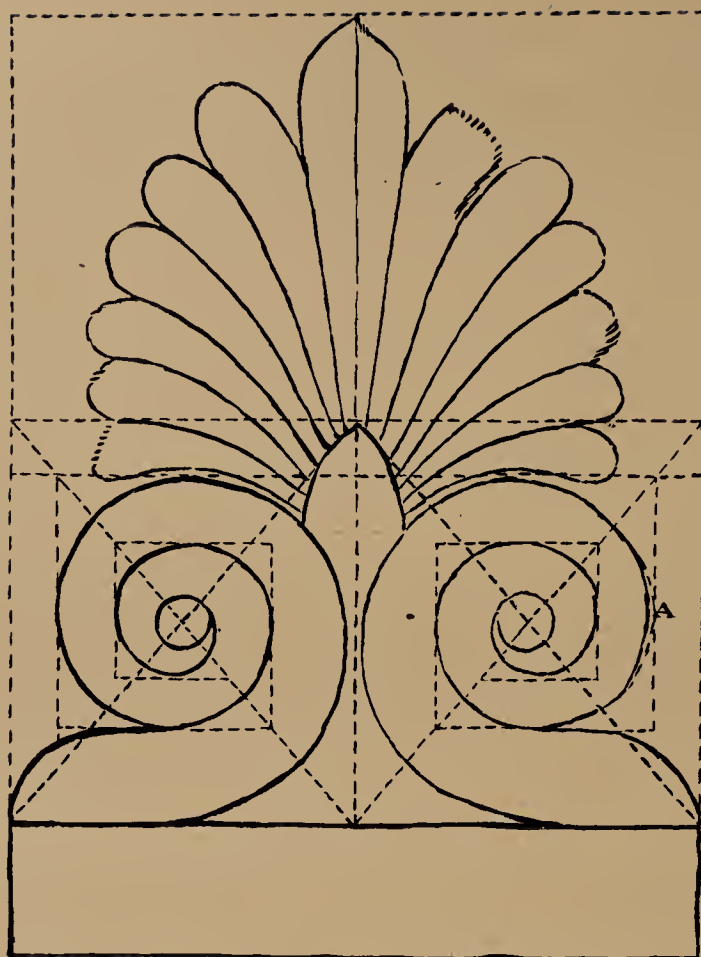


Figure 64.

CHAPTER NINE: THE ANTEFIX

THE eaves of the Parthenon roof were decorated by a double volute and palmette ornament called an antefix, one being placed at every alternate row of roof tiles.

Most of these have disappeared but there is one original in the Museum at Athens, one in the British Museum, and three on the building. These last are carved in the stones which support the lion head water spouts. There was one of these blind antefixes at each corner of the building. The one at the southeast corner has been destroyed. Two of the others, at the southwest and northwest corners, are reasonably accessible, and I obtained rubbings of them.

The spiral of this decoration affords an opportunity to say something about the Ionic volute. During the summer of 1902 I had several conversations with Mr. Penrose about the proportions of Greek buildings,

and I was his guest at a meeting of The Royal Institute of British Architects in November of that year when he read his well-known paper on the Ionic volute.*

It was his conviction that the best Greek volutes were very nearly logarithmic spirals, that is, they were close to being shell curves, but, as he pointed out in his paper, the shell curve will not satisfy all the requirements of the spiral as the Greeks made it. So keen was the eminent Englishman about the logarithmic spiral as a useful design curve that he invented an instrument for its mechanical construction. This he called the helicograph and exhibited it at the Crystal Palace Exhibition. It consisted of a sliding collar on a rod. One end of the rod was fastened to a drawing board. The sliding collar carried a toothed wheel and a pencil. The wheel could be set at any desired angle and as the rod was revolved, the sliding collar, forced by the toothed wheel, moved up and down, the pencil point thus tracing a logarithmic spiral of any required angle.

Mr. Penrose was convinced that the Greeks used a small cylinder of wood cut in steps at one end to hold a string. The cylinder was fastened to the eye of the volute and the string, as it was unwound, traced the spiral required.

This struck me as being an unworkmanlike proceeding, and I so told the author. He asked me to suggest something better. At the time I could not, but expressed my belief that the classic workmen were not likely to use other tools than those they employed in their trade. This incident aroused my curiosity and thenceforward I noticed carefully all the Greek spirals I found.

A short time after this I visited the British Museum and was at once struck by the original of an Ionic volute which belonged to the great temple at Ephesos. It was a custom of the ancients to finish the eyes of volutes carefully or to leave a hole for the insertion of a bronze plug or decoration. The eye of this volute, however, had not been finished and on it were the scratched lines dividing the area of the eye into eight parts and the actual marks of compass points in the stone, showing that the masons had set out the spiral with circle arcs. It was possible of course that some curious investigator had experimented with a compass on this head so I

* *Drawing and Construction of the Ionic Volute*, by Francis C. Penrose, F.R.S., Litt.D., D.C.L. Read before The Royal Institute of British Architects, November 17, 1902.

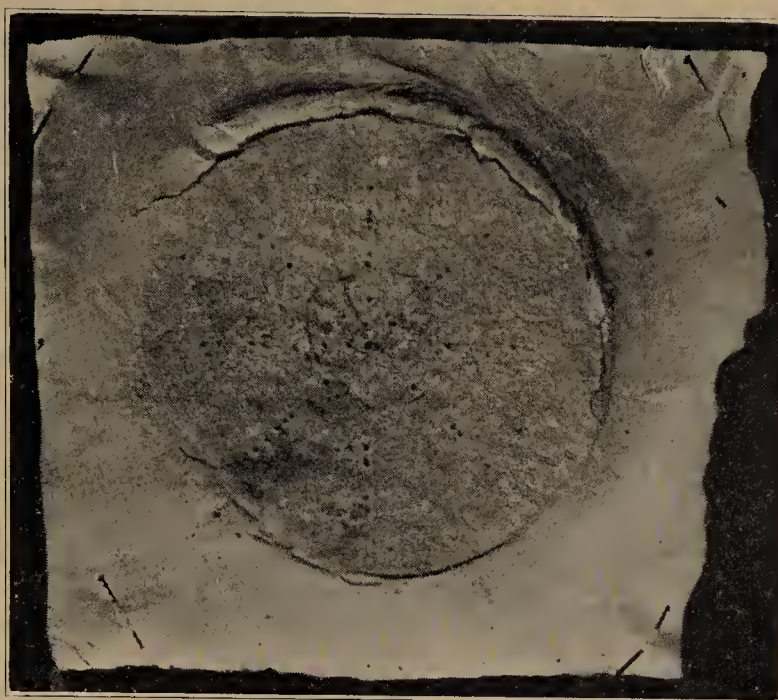


Figure 65.

Eye of a volute from the temple of Artemis at Ephesos.

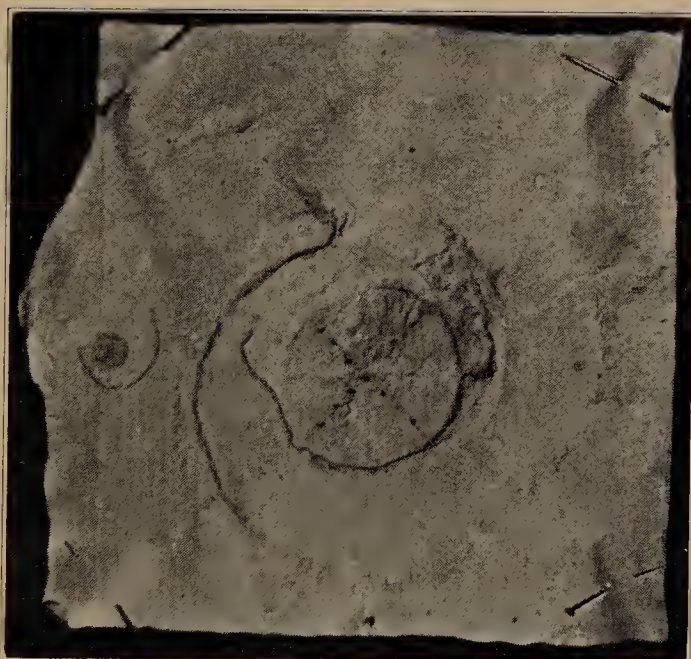


Figure 66.

Eye of a volute from Priene.

Such a spiral may be laid out by a workman with a carpenter's square.

Indeed this was how dynamic symmetry was discovered. I was struck by the fact that there must be rectangles of such proportions that their reciprocals would be even multiples of the rectangles themselves as 1:2:3:4:5, etc.; that is, that FG of figure 67 would be a multiple of AB.

These rectangles were soon fixed as having a square-root relationship; that is, the end-to-side proportions would be as unity to the square root of 2:3:4:5, etc.

It was thus that I hit upon the proportions discovered by the early Greeks when they found how to draw a square which would be any multiple of a square on a given linear base, which was much later described by Plato in the *Theætetus*. Squares on the ends and sides of these rectangles of square-root proportions are as unity to 2:3:4:5, etc.

To treat the Ionic volute exhaustively would require more space than is available in this book. I have done much work upon the subject and have found, as did Penrose, that the true Greek spiral is very nearly a logarithmic spiral. But the inverse proportion method of inspection tells us exactly how it deviates from the mathematical curve and how workmen may make it easily and accurately and proportion it in any desired way. Probably no volute is better for the purpose of illustrating this than that of the Parthenon antefix.

The most striking feature of the Parthenon antefix is that when the palmette is excluded the remaining proportion is that of a square and a root-five rectangle, AB of figure 68.

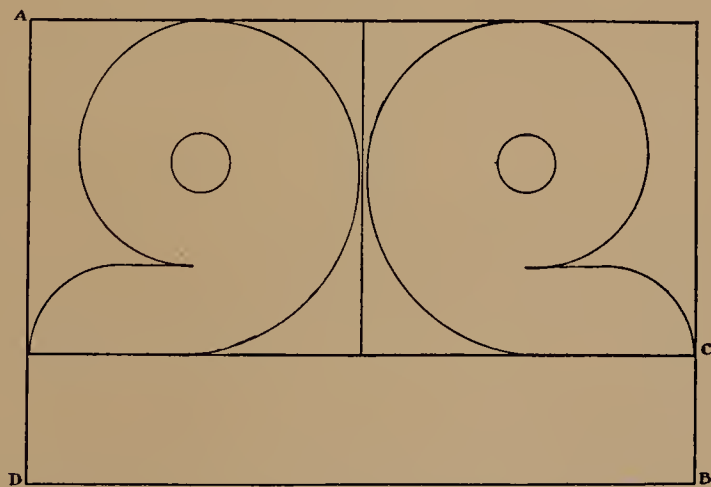


Figure 68.

The area occupied by the volute is composed of two squares, AC.
Consequently CD is composed of two squares and two 1.618 rectangles.

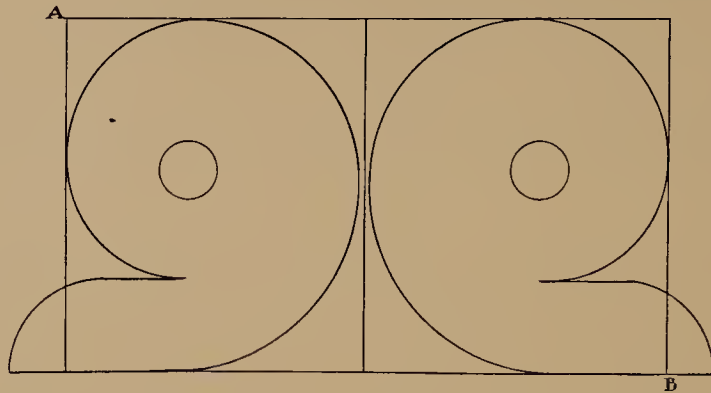


Figure 69.

The area AB of figure 69 is a similar figure to the rectangle of the façade projection, consequently each spiral is developed in a rectangle similar to one-half of the façade area.

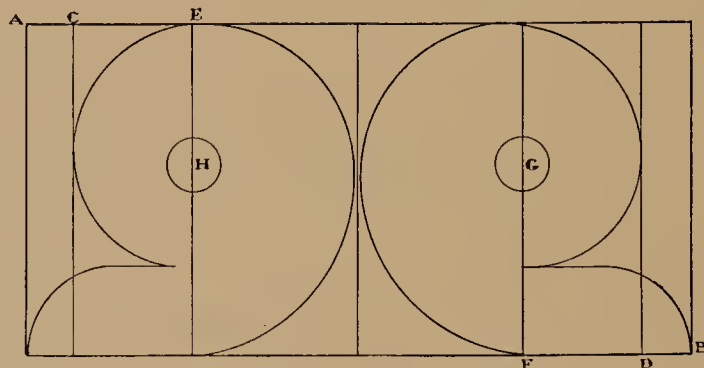


Figure 70.

AB of figure 70 is composed of two squares.
CD is a figure similar to the façade rectangle.
EF is a square two sides of which pass through the centers of the eyes H and G.
AB of figure 71 is a square.
CB is a figure similar to one-half of the façade area.
HI is the reciprocal of CB.
Diagonals to the two areas, CB and HI, cross each other at right angles

at E, the center of the eye of the volute. The lines BH, HC, CI, IJ, JK, KL, LM, MF, are the lines of the rectangular spiral.

It will be noticed that the spiral is irregular at G because it is slightly inside of the line BH; this is to allow for the shaft of the spear in the ornament.

From F, until it touches the circumference of the eye, the spiral moves

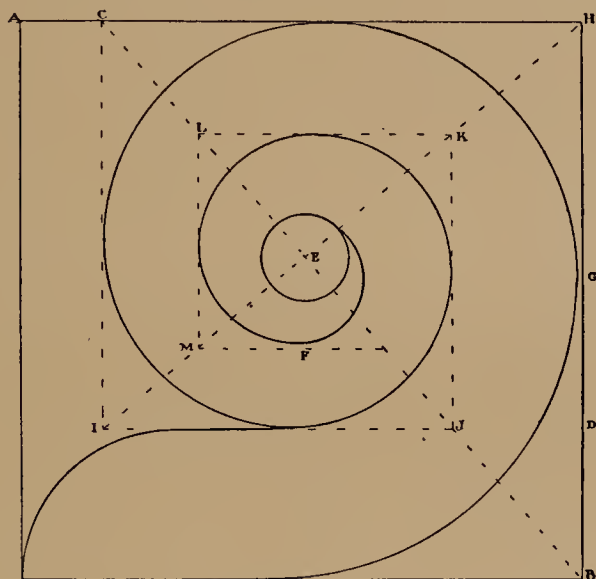


Figure 71.

too quickly toward the center. The curve is slightly irregular at F; that is, it does not quite touch the line MF.

Normally the spiral should wrap to the center as in *a* of figure 72, where it touches the eye at A.

In the actual spiral *b* it is brought up quickly from C to touch the circumference of the eye at D.

The designers evidently recognized that the normal spiral for this proportion would have too many windings for the size of the eye.

Consequently the actual volute is not strictly an illustration of continued proportion because of the variations from G to F and F to E. This irregularity is true for all the antefix volutes, and is thus characteristic of the Parthenon spiral.

Except for the irregularities noted the volute is very nearly a true shell spiral. That the Greeks thought of it as such is doubtful. In making it

they were following what appears a general practice of developing curves with the assistance of tangents.

The spiral of the antefix is a curve embodying the proportions of the façade of the building and the designers probably thought of it only in that light as the æsthetic motive was an old one.

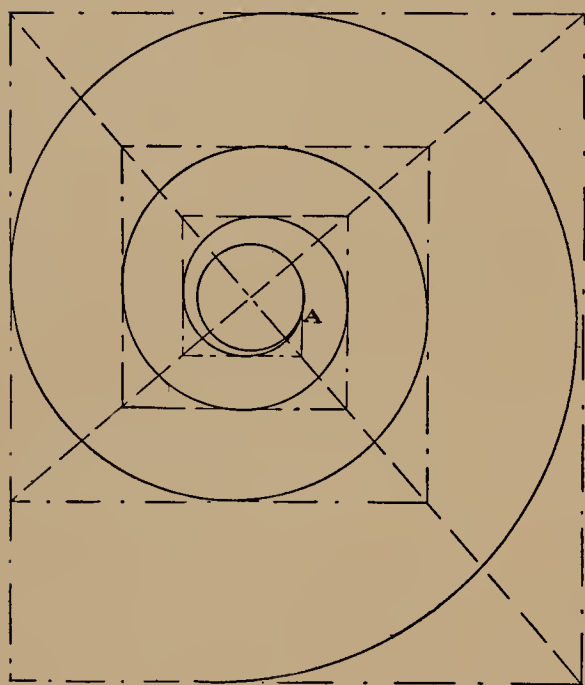


Figure 72 a.

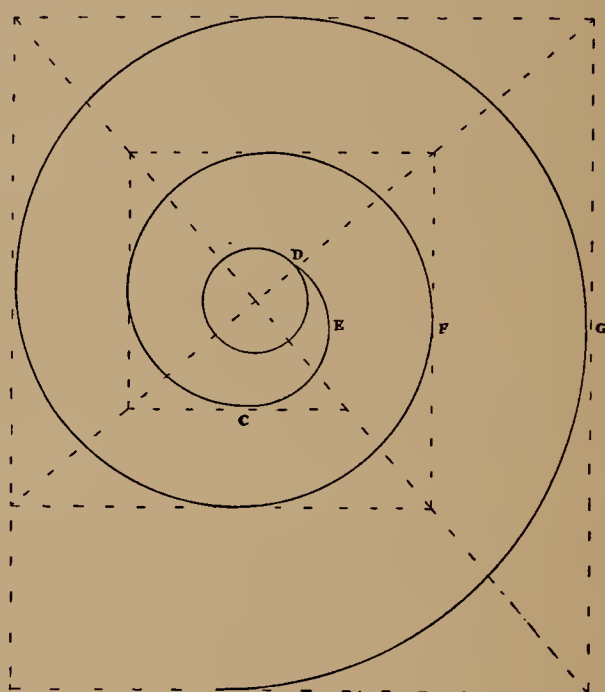


Figure 72 b.

a and *b*. How the real antefix volute would look.

The entire antefix has the following proportions:

AC of figure 73 is composed of a root-five rectangle and a half, or of three similar figures to the pediment and entablature projection. These three areas occupy a position on top of each other; see the Maiden's room of the cella.

CD equals a square and root-five rectangle of which CE is two squares. Consequently EB is composed of five squares and a .236 area. The reciprocal value is .191. EF is a similar figure to one-half of the temple façade. Diagonals to EF pass through the centers of the eyes of the volutes, G, H. The point of the arrow, I, is at the center of EF. Each volute is developed within an area similar to EF.

Behind the modern Greek museum on the Acropolis there are many

fragments of antefixes, some with the spiral section complete or nearly so. These fragments show that some of the antefixes were smaller than others. The spirals are all similar as may be proven by revolving rubbings of the smaller ones on the larger. Also the smaller ones have a less distance between the eye centers.

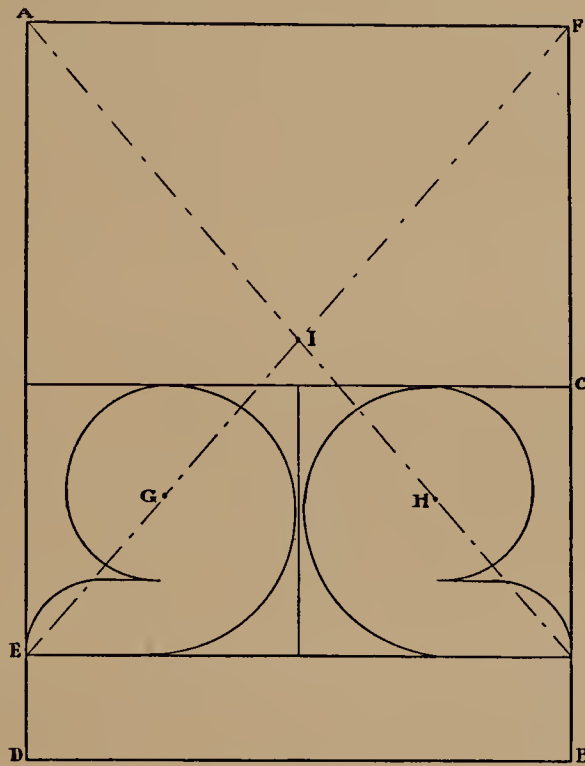


Figure 73.

The only explanation I can see is that the refinement curves of the flanks of the building were carried to the cornice and that the tops of the antefixes were on a straighter line, and, like the columns, those nearer the center were shorter. The volutes however show that the proportions were maintained.

AB of figure 74 is that part of the antefix which contains the volutes and the palmette ornament. It is similar to one-half of the façade rectangle. The line drawn horizontally through the point J divides AB into two equal parts. Each of these parts therefore is a similar figure to the façade rectangle.

KB is composed of two squares which contain the volutes.

DG, GL are diagonals to the two halves of DB.

These lines cut KP at E and M, consequently EF is a similar figure to DB or to the façade rectangle.

H and I are the centers of the eyes of the volutes.

CB is the blank .191 rectangle at the bottom of the antefix.

The ratio of OC is 1.3618. The AB ratio is 1.1708 or 1.3618 minus .191. .8541 or one-half of 1.7082 is the reciprocal of 1.17082.

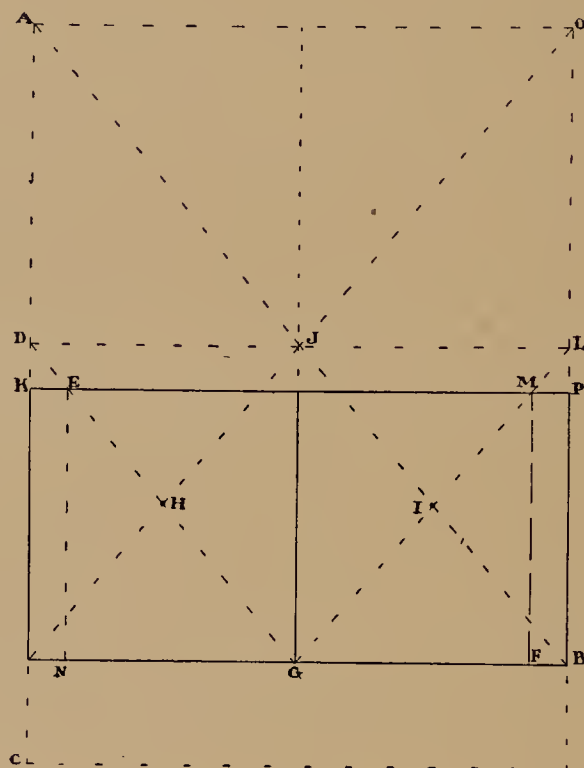


Figure 74.

AB of figure 75 is a square and a root-five area. AP is composed of two squares. CD is a similar figure to the façade rectangle.

CL is composed of two .618 rectangles.

ML is composed of two root-five rectangles.

EH is a square two sides of which pass through the centers of the eyes of the volutes.

GJ is a 2.618 rectangle.

MI, DJ are each composed of a .618 rectangle and a half (.618 plus .309 or .927).

MN and DB are each 2.618 areas, the reciprocal values of each being .382.

GN and HB, added, equal GJ.

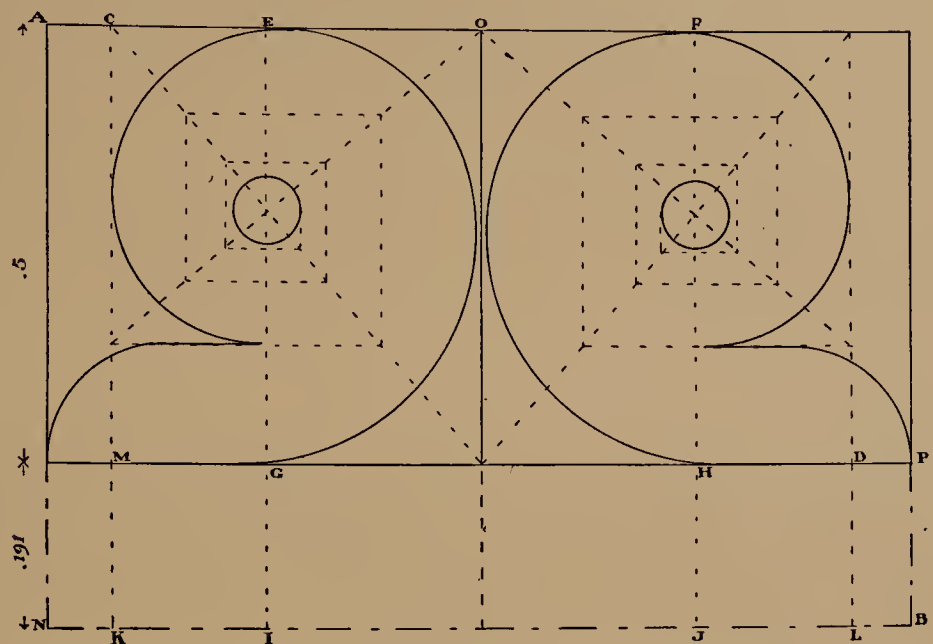


Figure 75.

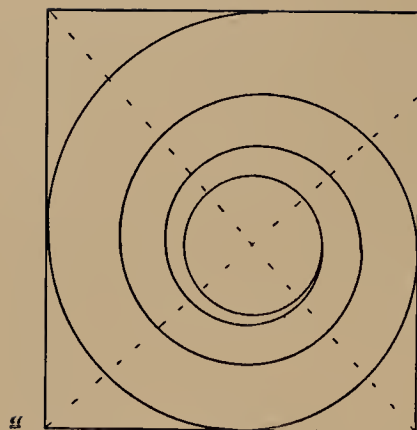


Figure 76 a.

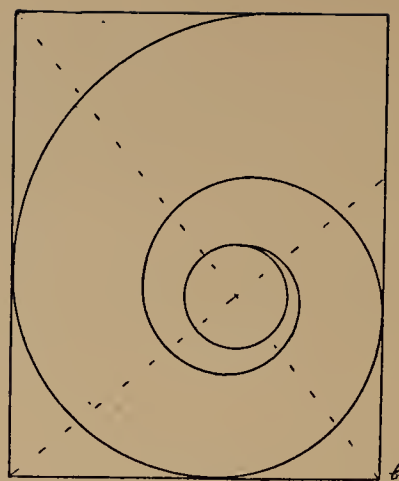


Figure 76 b.

CHAPTER TEN: THE GREEK SPIRAL

THE construction for a reciprocal rectangle suggests that there may be an ideal volute. The rectangles for Greek volutes vary but those which we recognize as the best approach a certain standard.

Slight consideration will make it clear that the volute for an Ionic capital must be contained in a rectangle which is not too nearly a square and not greater than a square and a quarter. If the rectangle is too close to a square it will have many windings or will necessitate a larger eye than the normal. A rectangle greater than a square and a quarter will produce a spiral of too few windings.

These points are brought out in figure 76. The 1.118 rectangle of *a* needs too large an eye. The 1.236 area of *b* has too great a separation between the windings.

Experiment has suggested that a rectangle with the ratio 1:1.191, or something close to it, makes a standard. The Parthenon antefix spiral is 1:1.17082. This makes too many windings as reference to figure 72 *a* will show.

The rectangular spiral made by a 1.191 rectangle seems to be the best because it possesses certain properties of balanced proportion.

AB of figure 77 is a 1.191 rectangle and CD is its reciprocal.

Their diagonals cross each other at right angles at E.

Through E draw the lines FH and GI parallel to the ends and sides of AB.

The areas CE, ED, LE, JE, etc., are similar to AB but are in position like CD.

BE, AE, KE, ME, etc., are similar to AB and are in position as AB. CF is composed of a similar figure to AB plus a reciprocal, AE, etc.

It will be noticed that the points N, J, K, L, M, etc., are exactly in the center of each corresponding rectangle as BE, CE, AE, DE, etc. This

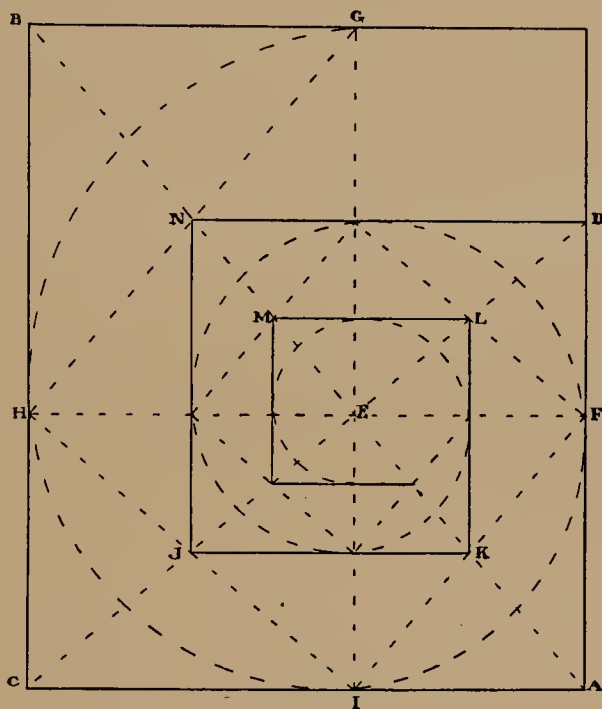


Figure 77.

First plan for an ideal volute.

centering furnishes the proportional balance referred to. A smaller or a larger rectangle will make the corners of the angular spiral fall on either side of these centers. This is the reason for suggesting that this rectangle is ideal for the volute of an Ionic capital.

The spiral curve may be fixed within the rectangle by quadrant or octant arcs of circles. The octant arcs are better, though quadrant arcs make a surprisingly smooth curve.

AB of figure 78 is a 1.191 rectangle. The fraction .191 represents an area similar to that of the blank rectangle at the bottom of the antefix. It is also similar to the abacus of the stele at Athens. (See Introduction.)

The diagonals AB and CD cross each other at right angles at S, which is the pole or center of the spiral.

Bisect the right angles ASD, ASC, CSB, BSD, and draw the lines JH and IK.

These lines, it will be noticed, are not parallel to either AC or CB.

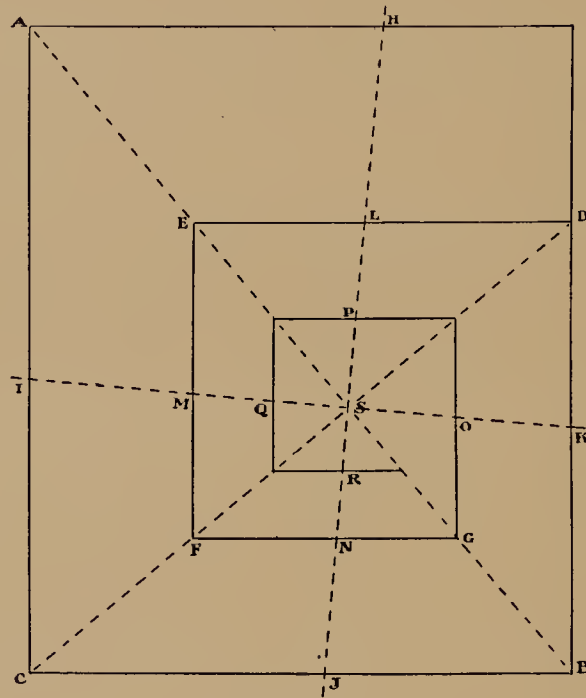


Figure 78.

If a circular eye were fixed at the pole its area would be divided into eight equal parts by the lines AB, CD, IK, and JH.

H, I, J, K, L, M, N, O, P, Q, R, will be tangent points for the curved spiral. That is, the curved spiral will touch the rectangular spiral at these points.

From the tangent point F of figure 79 draw FG, parallel to CB.

From I draw IH, parallel to CA.

From J the line JK, etc.

AFGE is a square as are also FCIH, IBJK, NJDL, etc., etc.

These areas form a whorl of squares about the pole M and their sides will naturally be the radii for the quadrant circle arcs for the curved spiral: GHKN, etc., will be the centers for the arcs and will lie on the lines AB and CD.

If octant arcs are used for fixing the curved spiral additional points will be found on the lines FJ and IE.

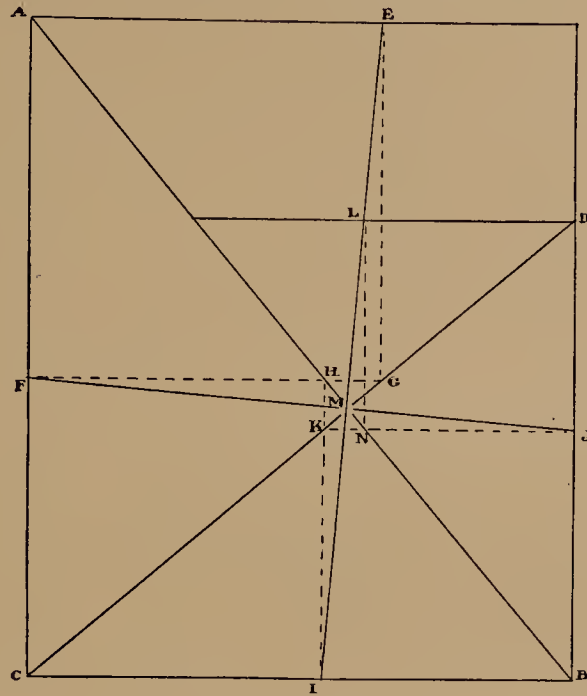


Figure 79.

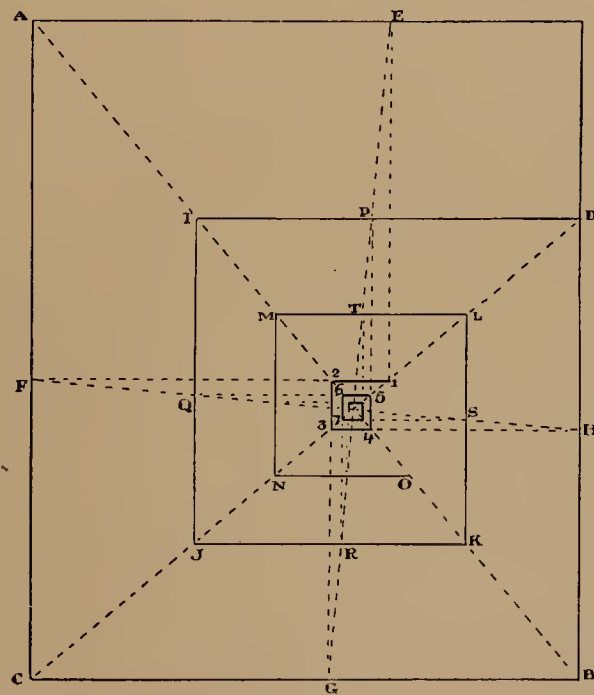


Figure 80.

When the radii for the quadrant arcs are drawn as 1EF, 2FG, 3GH, 4HP, 5PQ, 6QR, 7RS, 8ST, etc., the centers make a rectangular spiral around the center of the eye. This may be called an involute of the major rectangular spiral. (See figure 80.)

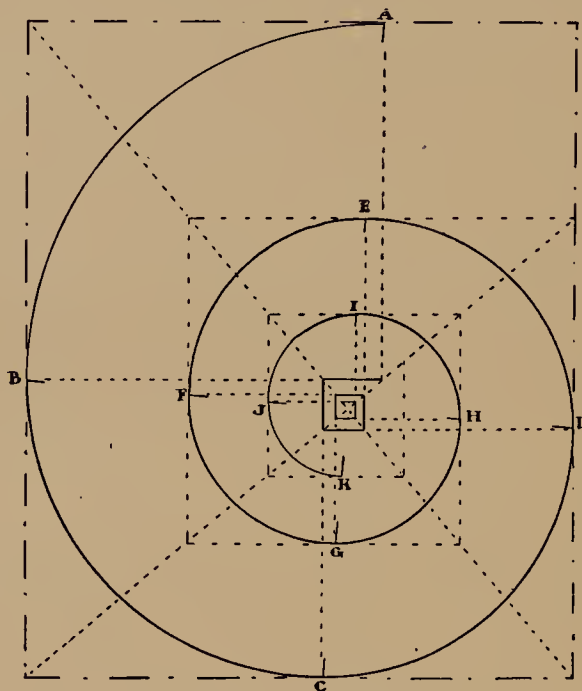


Figure 81.

Figure 81 exhibits the curved spiral of a 1.191 rectangle. The same principle of construction applies to any rectangle.

The proportion of the rectangle will decide whether the spiral will wind tightly or loosely. This rectangle, however, satisfies the requirements of the Ionic volute better than any other. With this principle, elliptical or any type of spiral may be drawn easily and accurately.

The rectangular spiral is quickly made and the designer may judge from it how the curved spiral will look.

The points where the curved spiral is tangent to the rectangular spiral are A, B, C, D, E, F, G, H, I, J, K, etc.

EC of figure 82 is a reciprocal of the rectangle AD and the two diagonal lines cross each other at right angles at B.

The lines FG and HI bisect the angles ABC, CBD, DBE, and EBA.

HJFA is always a square whatever rectangle is used.

As a dynamic rectangle plus a square is found so frequently in Greek

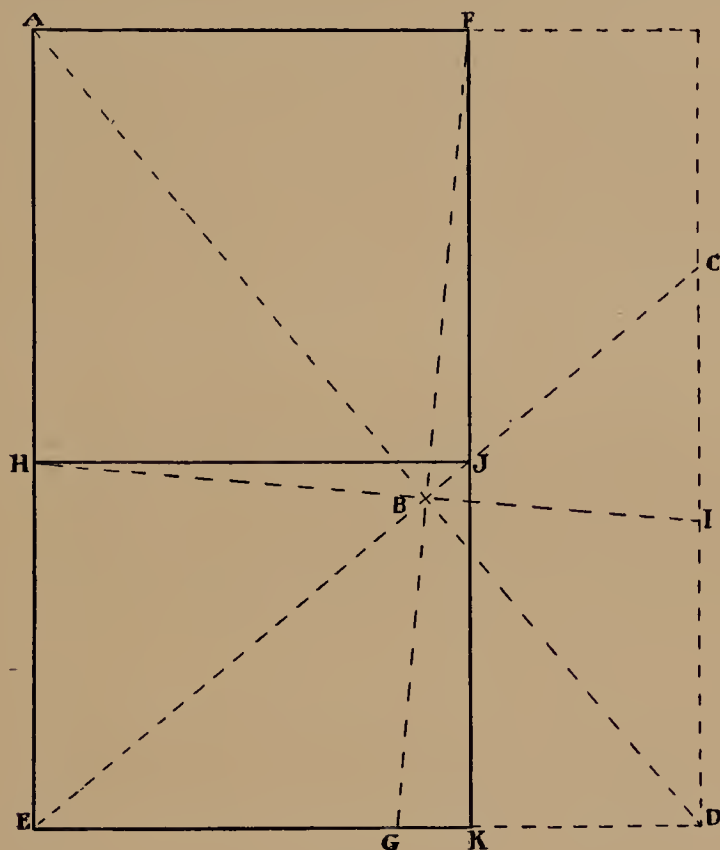


Figure 82.

The closer the original rectangle is to a square the more perpendicular will be the line GF.

AB of figure 83 is a 1.618 rectangle and CD is a reciprocal.

The two diagonals AB and CD cross at right angles at E.

FG and HI, at right angles through E, bisect the angles AED, DEB, BEC, and CEA.

F is a corner of the parent rectangle.

H is a corner of the square AD.

The coincidence of H and F with corners of the square AD exhibits a

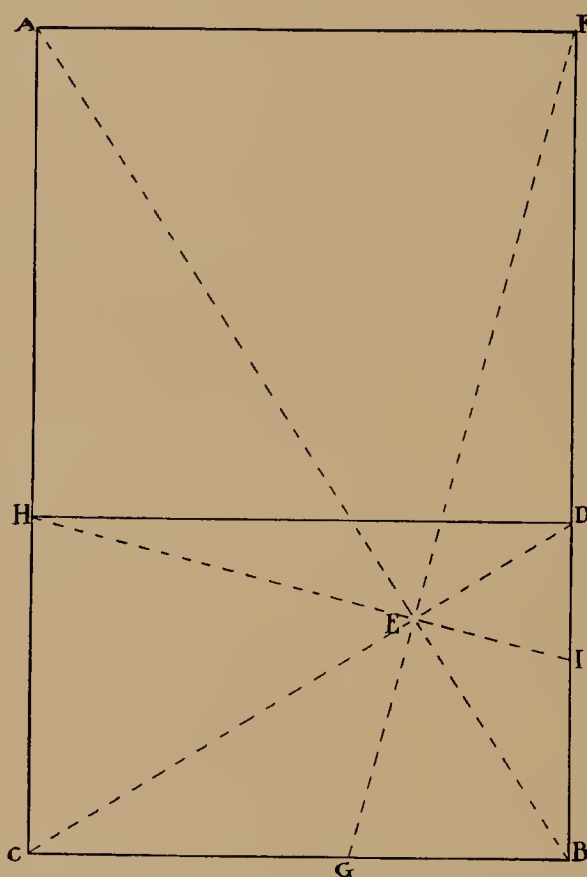


Figure 83.

balancing property of this rectangle and is one of the reasons why I called it the "rectangle of the whirling squares." (See *Dynamic Symmetry: The Greek Vase*.)

AB of figure 84 is a root-five rectangle.

CD is a diagonal to a reciprocal.

EF and GH are bisectors of the angles AJD, DJB, BJC, and CJA.

AK is a square and CK a similar shape to AB.

AI is a square plus a root-five rectangle.

(See the ground plan of the Parthenon.)

The “squeeze” of the eye from the temple at Ephesos shows compass marks on diagonals of a real or quasi octagon. A construction for octagonal divisions of the eye circle is shown in figure 85. The centers for the arcs are at 1, A, 2, B, 3, C, 4, D, 5, E, 6, F, 7, G, etc., and these points are found on the lines WX and YZ where they cross the rectangular spiral.

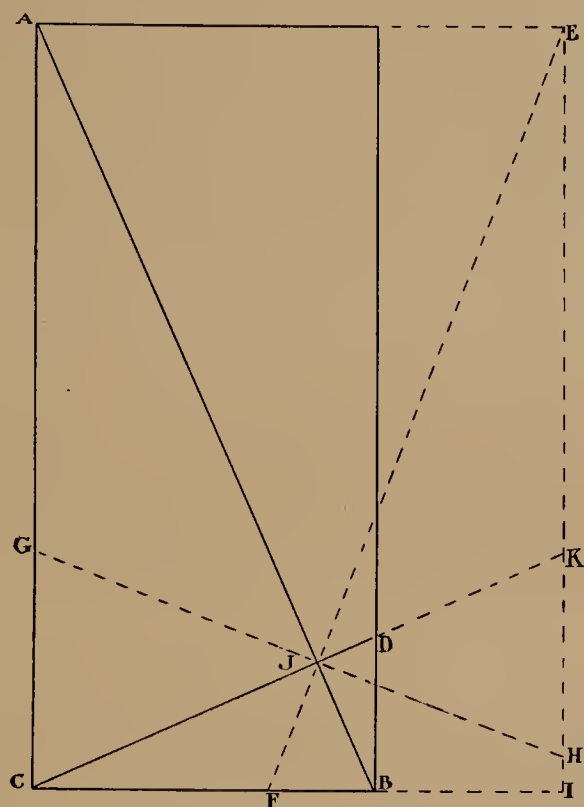


Figure 84.

The clear line curve of the figure is made by quadrant arcs with centers at the corners of the squares at 1, 2, 3, 4, 5, 6, 7, etc. The effect of the octant arcs is shown by the dotted lines from H to I, J to K, L to M, N to O, P to Q, etc.

Octant arcs make a fuller curve which is, undoubtedly, preferable for a very large spiral such as the one from Ephesos.

Further illustration of the peculiar balance of a rectangular spiral in a 1.191 area is illustrated in figure 86. It will be remembered that the corners of a rectangular spiral in a 1.191 area touched the centers of the similar figures which composed the generating rectangle. (See figure 77.) In the present construction the similar figures are discarded for the series

of squares which occupy a whorled position around the center of the spiral. The squares are ML, LO, OR, RS, SV, VW, etc. The rectangular spiral touches the diagonals of these squares at E, F, G, H, I, J, K, etc.

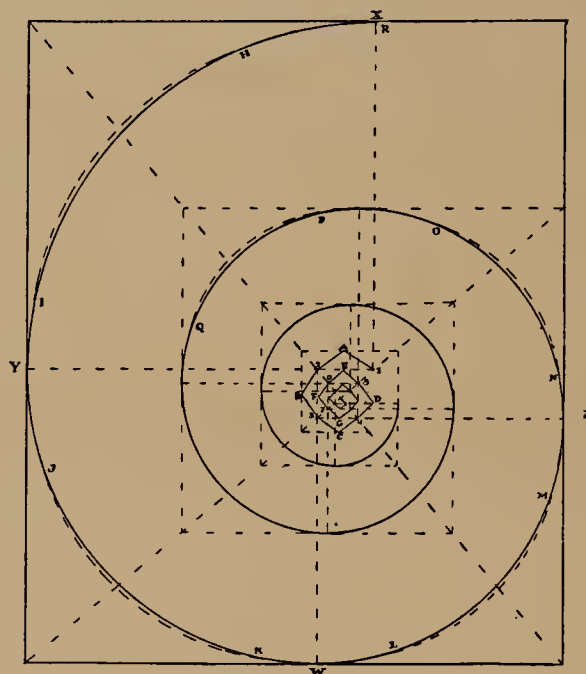


Figure 85.

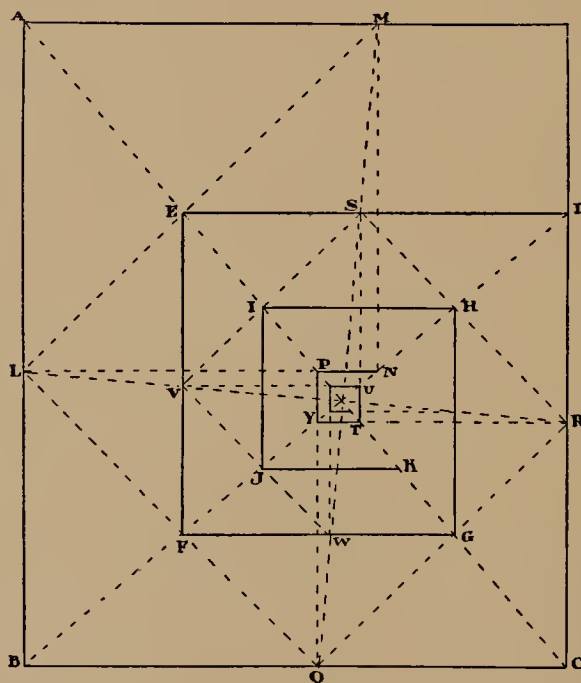


Figure 86.

Hence the rectangular spiral E, F, G, H, I, J, K, etc., is exactly contained in the rectangular spiral M, L, O, R, S, V, W, etc.

The corners of the whorled squares are at N, P, Y, T, U, etc.

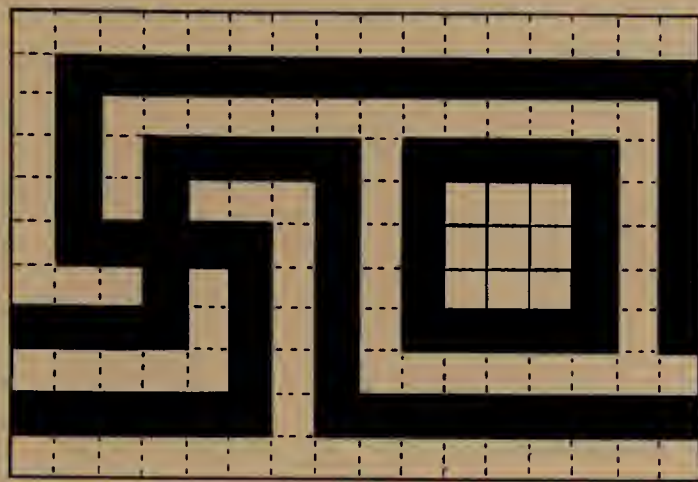


Figure 87.

Single meander pattern from the cornice of the Parthenon at Athens. Rectangle 1 : 1.472. Divided horizontally into eleven parts and vertically into sixteen parts.

THE GREEK MEANDER

It has probably occurred to the reader that the rectangular spiral resembles closely the celebrated Greek key, fret, or meander pattern.

There were two meander patterns on the Parthenon of unusual distinction, a compound one over the Panathenaic frieze and a simple one under the cornice.

The surface of the marble seems to have been covered with a coat of white, probably gypsum or some mixture with that in it. On the surface of this, vertical and horizontal lines were drawn with some sort of a point. This made a web or mesh and from this background the pattern was picked out. Several blocks of marble belonging to the courses on which these patterns were fixed are now lying on the ground near the building and the vertical and horizontal scratched lines are more or less visible. Some of the paint or gold leaf or whatever was used to define the pattern has been affected by the long exposure more than the rest, the result being that the pattern, in many places, is rather clearly brought out by weather etching.

Modern designers make a Greek fret by ruling vertical and horizontal lines to form a mesh of small squares and pick out the pattern on this. When I first saw these stones with the remains of the Greek ornament on them I thought that in one way at least modern and ancient practice agreed.

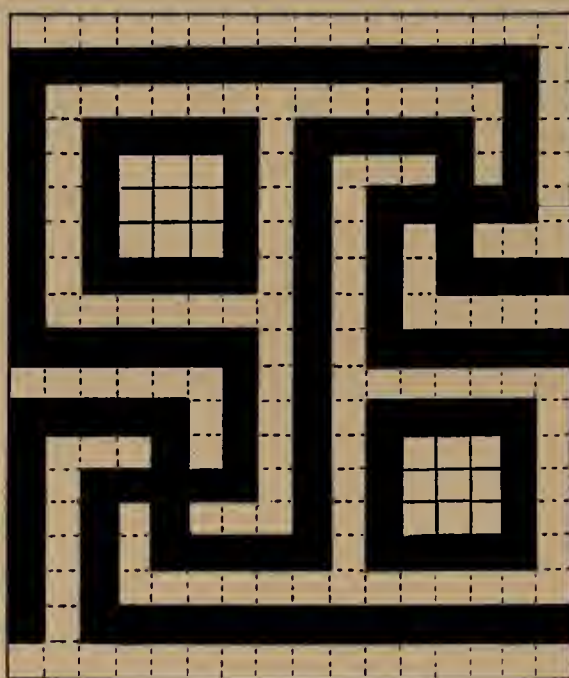


Figure 88.

Double meander pattern over the Panathenaic frieze of the Parthenon. Rectangle 1 : 1.18. Divided horizontally into nineteen parts and vertically into sixteen parts.

I counted the vertical and horizontal lines for a complete pattern, laid out a mesh of squares for the required height and picked out the decorative unit. When the result was compared with the Greek original it did not fit the unit length. Before I could make a fit, a dynamic area had to be made and divided into a rectangular mesh. The result was not composed of squares but of rectangles. Rather close to squares it is true but certainly not squares. Probably a real Greek meander has not been made since classic days. The unaided or untrained eye can scarcely tell the rectangle from the square but the difference perhaps is that which dis-

tinguishes Greek masterfulness from modern clumsiness, *dynamic* from *static* symmetry.

For the single meander pattern under the cornice a rectangle composed of a square and two .236 rectangles had to be used. This area was divided vertically into sixteen parts and horizontally into eleven parts and the pattern picked out from this mesh of seeming squares. The ratio is 1 : 1.472.

The compound meander pattern over the Panathenaic frieze demanded a rectangle composed of two squares plus a .618 area; that is, .5 plus .618. This rectangle is a similar figure to the area of the façade, without the pediment, when a square is subtracted from it. It might also be fixed as a rectangle composed of five .236 areas. The ratio is 1 : 1.18.

This area is divided horizontally into nineteen parts and vertically into sixteen parts, and the pattern picked out as in figure 88.

The construction for a volute brings out rather clearly some fundamental ideas relative to divisions of rectangles for purposes of design.

CHAPTER ELEVEN: ANOTHER TEMPLE BY THE ARCHITECT OF THE PARTHENON

NEAR the bleak and lonely summit of Mount Kotylion, at Bassæ in Arcadia, are the remains of the Doric temple of Apollo Epikourios which was designed by Iktinos, the architect of the Parthenon.

Pausanias fixes the date for the building of this temple after the Parthenon was completed when the fame of the architect had been established. There are some modern authorities who are inclined to an earlier date. Judging solely from the nature of the proportions of the design I believe Pausanias was right.

The building is interesting because it has no curve refinements such as are found in the Parthenon. It is a straight line design throughout—even the columns are without entasis. Penrose visited the site but when he found no curves in plan or elevation he, apparently, lost interest and published nothing. The English architect Cockerell made a beautiful drawing of the temple but the scanty measurements he supplied are not borne out by the facts. The French architect Blouet, who was head of the Scientific Expedition to the Morea in 1837, published a few measurements. These agree with those I was compelled to take when I visited the temple site in January, 1921. This was after the Greek Archæological Society had made rather extensive repairs.

The building was constructed of local stone which contains a good deal of flint. The softer stone has been much weather etched but the flint seems to be as the masons left it. There are many irregularities in the building as it stands which seem to be due to earthquake disturbance. There is also a sinking of made ground under the west flank. The position of the building is peculiar in that it was orientated north and south. The statue room, however, had an east door in the flank wall of the cella. Because of the irregularities I am of the opinion that an error of two centimeters, plus or minus, may be allowed in long measurements.

Although all the columns of the peristyle are still in position, excepting the one at the southeast corner, I do not believe that anything reliable can be done with the elevation plans; therefore I have confined my attention to the ground plan and its subdivisions as they are fixed by members now in place.



Figure 89.

The Doric Temple of Apollo Epikourios at Bassæ designed by Iktinos
of Parthenon fame.

The overall plan, as fixed by the levelling course, the euthynteria, is composed of four .618 rectangles.

The top step rectangle is composed of two squares and a .618 rectangle.

The cella exterior, fixed by the step, is composed of two 1.618 rectangles.

The cella interior plan is composed of four .618 areas.

The relation of this cella interior plan to that of the overall plan is 1 : 2.36. (See the cella interior of the Parthenon.)

The rectangle made by the angle column centers of the peristyle is composed of a square and four root-five rectangles.

The floor plan of the statue room is composed of a square and two .236 rectangles. Great accuracy here is hardly possible.

The sunken floor of the pronaos is a 1.618 rectangle.

The sunken floor of the opisthodomos is a rectangle composed of two squares plus a similar figure to the rectangle made by the top step of the temple.

The rectangle made by the floor of the nave is composed of two squares and two root-five rectangles.

The columns surrounding the nave are placed by root-five rectangles while the column bases are fixed, as regards their diameters, by .236 rectangles.

The ground plan of the temple at Bassæ supplies an interesting illustration of an ancient method of surveying. This operation was carried out by harpedonaptæ, literally "rope fasteners." Cantor, the German mathematical historian has shown, from inscriptions on the Egyptian temples, that the duty of these "rope fasteners" consisted in the orientation of the buildings by reference to the constellation of the Great Bear. "The meridian being thus found, the line at right angles to it was probably determined by the construction of a triangle with ropes measuring three, four, and five units respectively." The operation of rope stretching, according to Cantor "was one of unknown antiquity, being noticed in a record of the time of Amenemhat I, preserved in the Berlin Museum." (See *Allman*, p. 218.) A quotation from the Greek mathematician and philosopher, Democritus, has survived, "I have wandered over a larger portion of the earth than any man of my time, enquiring about things most remote; I have observed very many climates and lands, and have listened to very many learned men; but no one has ever yet surpassed me in the construction of lines with demonstration; no, not even the Egyptian har-

pedonaptæ as they are called, with whom I lived five years in all, in a foreign land." (*Allman*, p. 80.)

The passage indicates that these surveyors were known as expert geometers, *i.e.*, "earth measurers." We know that the Greeks were familiar with the idea of fixing right angles by numbers as were also the early Hindus since their formulæ for this purpose have survived. Engineers in the field today, when they are without instruments, use the same methods.

The ground plan of the Bassæ temple would be fixed by this old method as follows:

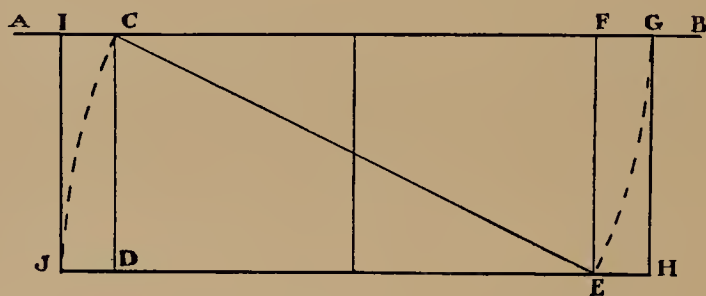


Figure 90.

Let AB be an orientating line; in this case fixed by a sighting method due north and south.

Upon this line a right-angled triangle would be defined as FCE and the two squares CE established. CD would be the selected width for the temple.

A man stationed at C and another at E would hold the rope CE taut.

The man at C would hold the rope, probably with the aid of a stake, while the one at E walked to G, where another stake would be driven.

DG is a root-five rectangle.

While the last-mentioned man retraced his steps and stood at E, and used a stake, the other, at C, would walk to J, thus fixing the area GJ.

JC and EG are each .236 rectangles, and the entire area JG would have the ratio of root five plus .236, or 2.472, that is, $.618 \times 4$.

Note. From measurements of the Greek temple of Concordia at Girgenti, Sicily, made by Perrot and Chipiez, the French mathematician M. Paul Tannery found that the rectangle made by the angle column centers was composed of four .618 rectangles, that is, it had the ratio 2.472.

If a radius of a circle equals 1, the side of a decagon inscribed in it equals .618. Since the publication of Tannery's observations there have been attempts made to account for the proportions of Greek temples in terms of the relation of a side of a decagon to a radius, particularly by the Germans. The process has been called by them "circle geometry."

However, the German archæologists Koldewey and Puchstein have measured the Concordia temple since the time of Perrot and Chipiez with different results. Thus Tannery's quadruple .618 scheme is not proven. It would be interesting if the Bassæ proportion of 2.472 could be established in the temple of Concordia, but, from the general character of Koldewey and Puchstein's measurements of many other Greek temples I am inclined to favor the German results.

ARITHMETICAL PROOF FOR THE BASSÆ TEMPLE

OVERALL plan rectangle $16.09 \pm$ by $39.804 \pm$ meters. Ratio 2.472 equals $.618 \times 4$ or 2.236 plus .236. Error .025.

The top step rectangle is $14.626 \pm$ by $38.324 \pm$ meters. Ratio 2.618. Error .012.

The outside length of the cella is $28.07 \pm$ and the width $8.66 \pm$ meters. Ratio 3.236 or 1.618×2 . The reciprocal of 3.236 is .309. $28.07 \times .309$ equals 8.673. Error .013.

The length of the cella interior is $16.864 \pm$, width $6.822 \pm$. Ratio 2.472 (same as the overall plan).

Length of the cella interior multiplied by 2.36 equals the length of the overall plan.

Width of the cella interior multiplied by 2.36 equals the width of the overall plan.

The rectangle made by the angle column centers is $13.250 \pm$ by $36.948 \pm$ meters. Ratio 2.788854 ($.4472 \times 4$ equals 1.788854; plus 1 equals the ratio).

Floor plan of the statue room, $4.634 \pm$ by $6.822 \pm$ meters. Ratio 1.472 ($.472$ equals $.236 \times 2$). Error .001.

The sunken rectangle of the pronaos, $4.061 \pm$ by $6.564 \pm$ meters. Ratio 1.618. Error .005.

The sunken rectangle of the opisthodomos, $2.756 \pm$ by $6.564 \pm$. Ratio 2.382. Error .0009.

Relation between the last two rectangles 1 : 1.472.

The rectangle of the floor of the nave, $3.655 \pm$ by $10.570 \pm$, is composed of two squares and two root-five rectangles. Ratio 2.8944. Error .01.

THE TEMPLE AT SUNIUM

THE remains of the Doric temple at Sunium, a few miles to the south of Athens, are in bad condition. They were measured by Doerpfeld and by the Greek Archæological Society. The results of the two are almost identical. Mr. Orlandos, for the society, found a minute difference in the width as given by Doerpfeld.

The plan, including the euthynteria, is composed of three .618 rectangles and a half.

(Length $32.87 \pm$, width $15.20 \pm$ meters. Ratio 2.163; equals $.618 \times 3\frac{1}{2}$. 2.163×15.20 equals 32.8776. Error .0076.)

The top step rectangle is composed of two squares plus one-half of .618.

(Length $31.15 \pm$, width $13.48 \pm$ meters. Ratio 2.309. Error .025.)

The bottom step length is 32.67, width 15.00 meters. The area is composed of four root-five rectangles, and a 1.618 area ($1.618 + .559$).

(The ratio is 2.177. This, multiplied by 15.00 equals 32.655. Error .015.)

The length of the cella is 20.97, the width 8.32 meters. The area is composed of squares and root-five rectangles.

($.691 \times 3$) + .4472.

Considering the ruined condition of the building it is difficult to imagine closer agreement.

S	$\sqrt{5}$	.618
	$\sqrt{5}$	.618
	$\sqrt{5}$	.618
	$\sqrt{5}$	.618

Figure 91.
Ground plan of Ægina Temple.

CHAPTER TWELVE: THE DORIC TEMPLES AT ÆGINA AND OLYMPIA

THE Doric temple at Ægina, a building considerably older than the Parthenon, was measured by Fiechter, and his results published in Furtwängler, *Ægina, das Heiligtum der Aphaia*. He found the overall rectangle of the ground plan to be 15.53 by 30.50 meters.

This rectangle is composed of squares and .618 rectangles the primary arrangement of which is like the diagram, figure 91. There are many other combinations of the proportion possible but the one of the diagram seems to me the best. The error is about four millimeters.

The rectangle made by the cella, including the step, is composed of four squares and four root-five rectangles as in figure 92. Error .027.

A	C			
	$\sqrt{5}$	$\sqrt{5}$	$\sqrt{5}$	$\sqrt{5}$
	S	S	S	S
	B			

Figure 92.

The floor of the interior of the cella makes a rectangle composed of three squares and three root-five areas, *i.e.*, a similar shape to CB of the diagram. Error .009.

The intercolumniation strips of the cella interior, as fixed by column centers across the nave, are each composed of a square plus a square and a root-five rectangle. Error .005.

The ground plan of the nave is a 4.236 rectangle, reciprocal .236. This proportion is similar to that of an intercolumniation strip of the Parthenon cella.

The rectangle of the cella, including the column bases, is a similar figure to the cella (outside) of the Zeus temple at Olympia. The area is composed of two squares and two .427 rectangles. (See one-fourth of the Parthenon façade.) This rectangle may also be regarded as root five plus .618. Error .010.

The rectangle of the statue base is composed of .618 plus $.236 \times 3$. The latter shape is similar to the rectangle of the façade of the Parthenon.

THE ZEUS TEMPLE AT OLYMPIA

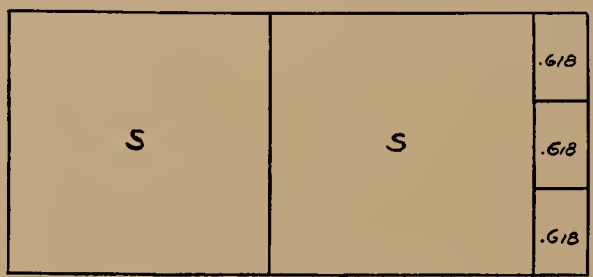


Figure 93.

The ground plan proportion of the Zeus Temple.

THE ground plan of the temple, including the first levelling course, is a rectangle composed of two squares and one-third of a .618 rectangle. The proportion would be perfect if eight millimeters were added to Doerpfeld's measured width. However, the flank lengths are very irregular, there being a difference of over forty-five centimeters between them. This very great discrepancy is undoubtedly due to earthquake disturbance, so the German measurements must be accepted with caution. A mean between the two flanks is probably that of a line through the center of the stylobate. This length is fairly close to the Doerpfeld results.

The rectangle of the cella, including the step, is composed of a root-five rectangle plus a .618 area. (See the same proportion in the temple at Ægina.)

The rectangle made by the peristyle column centers is composed of two squares plus a root-five rectangle. Error .012.

The area defined by the top step of the temple is composed of four root-five rectangles plus a square and two root-five areas.

The bottom step rectangle is composed of two squares and two root-five rectangles, the latter making a similar figure to the pediment and epistyle projection of the Parthenon.

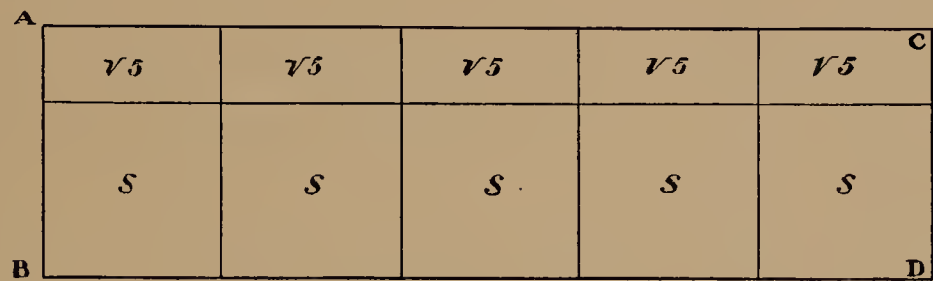


Figure 94.

From the remains of the temple as they now exist it is plain that special emphasis was placed upon that part of the cella interior defined by the column centers, as also appears to have been done in the cella of the Parthenon. In the Zeus temple part of this space was carefully separated from the rest of the interior by a stone barrier which excluded visitors from the immediate neighborhood of the statue of the god. There are marks on the pavement of the nave in the Parthenon which indicate that a metal railing served this purpose at Athens.

The area of the cella interior defined by the column centers, in the Zeus temple, is composed of five squares and five root-five rectangles arranged as in the diagram, figure 94.

AB and CD are fixed by the end walls of the cella, so that the two rows of columns, centered on the lines AC and BD, constitute a passageway through the center of the sacred chamber.

The width of this area is 8.32 and the length 28.74 meters. The pattern of squares and root-five rectangles of the diagram have the ratio of 3.455, *i.e.*, $.691 \times 5$. Error .002.

The sacred precinct of the god, as defined by the barrier mentioned, was an enclosure for the Phidian masterpiece of Zeus. The floor area of this enclosure has the same proportion as the similar area surrounding the statue of Athena at Athens. Each area is a 2.36 rectangle.

According to Doerpfeld's measurements the 2.36 area of the Zeus statue is 8.32 by 19.60 meters. The error is nearly two centimeters but we are unable to say exactly how this area was fixed, whether by the center line of the barrier or by either of its two sides. The interior of the cella is in much of a mess and it is difficult to obtain extremely accurate measurements as will be understood from the varying lengths of the flanks of the temple mentioned above.

ARITHMETICAL PROOF FOR THE ÆGINA TEMPLE

WIDTH of the ground plan 15.53; length 30.50 meters. Ratio 1.9635. Error .004. This is an arrangement of squares and .618 rectangles, *i.e.*, a square and a half plus .618 and a half.

Width of the cella, including the step, 8.27; length 22.785 meters. Ratio 2.764, which equals $.691 \times 4$. Error .027.

Interior of the cella 6.38 by 1.3245 meters. Ratio 2.073 or $.691 \times 3$. Error .009.

Intercolumniation strip of the cella, center to center of the columns, 2.28 by 3.85 meters. Ratio 1.691, *i.e.*, a square plus a square and a root-five rectangle. Error .005.

Ground plan of the nave 3.05 by 12.945 meters. Ratio 4.236. Error .025.

Rectangle of the cella floor, including the column base supports, 4.65 by 13.245 meters. Ratio 2.8541. This equals root five, 2.236, plus .618. Error .010.

Rectangle of the statue base .92 by 1.22 meters. Ratio 1.326. This is composed of .618 plus .7082, or $.236 \times 3$.

ARITHMETICAL PROOF FOR THE ZEUS TEMPLE AT OLYMPIA

WIDTH of the ground plan, including the euthynteria, 30.20; length 66.64 meters. Ratio 2.206. This is two squares and one-third of .618.

Rectangle made by the cella steps, 16.39 by 46.84 meters. Ratio 2.8541. Composed of 2.236 plus .618. Error .021.

Rectangle of the peristyle column centers, 25.20 by 61.70 meters. Ratio 2.4472. (Two squares and a root-five rectangle.) Error .012.

Rectangle of the top step of the temple 27.68 by 64.12 meters. Ratio

2.3166. (Four root-five areas plus a square and two root-five shapes. Error .0002.

Rectangle of the bottom step of the temple 29.76 by 66.20 meters. Ratio 2.2236. (Two squares and two root-five rectangles.) Error .026.

Area of the floor of the cella interior, defined by the column centers, and full length 8.32 by 28.74 meters. Ratio 3.455, *i.e.*, $.691 \times 5$; Error .002.

Area fixed by the barrier around the statue, 8.32 by 19.60 meters. Ratio 2.36. Error .018.

APPENDIX: NOTES

NOTE I.

IN mathematical language *δύναμις* signifies power, especially the second power or square. In the passage [Plato's *Theætetus*], however, the word seems not to be used steadily in the same signification, and in 148A it certainly means 'root.' M. Paul Tannery considers that the present text of Plato is corrupt, and that in it *δύναμις* (power) should be replaced throughout by *δυναμένη* (root). Professor Campbell (*Theætetus of Plato*, p. 21, note.) thinks that 'it is not clear that in Plato's time this point of terminology was fixed.' But, on the other hand, J. Barthelemy Saint-Hilaire believes that the expression *δύναμις* was probably invented by the Pythagoreans (*Metaphysique d'Aristote*, tom. II, p. 156, n. 16.). In support of this view it may be noticed that the term *δυνάμει* is used in its proper signification throughout the oldest fragment of Greek geometry—that handed down by Simplicius from the *History of Geometry* of Eudemus on the quadrature of the lunes (see pp. 67-70; and, for the revised Greek text, *Simplicii in Aristotelis physicorum libros quatuor priores commentaria*, ed. H. Diels, pp. 61-68, Berlin, 1882)—and is so used, for the most part, in paragraphs which, according to the *criterion* laid down in P. 72, n. 45, must be regarded as genuine. Now since Eudemus, in this fragment, gives an analysis of the work of Hippocrates, and, moreover, frequently refers to him by name, it is probable that, in parts at least, he quoted the work on lunes textually, and that the word *δυνάμει*, which occurs throughout, must have been used by Hippocrates, who we know was connected with the Pythagoreans. On the whole then it seems to me probable that Plato had not fully grasped the distinction between the terms *δύναμις*; and *δυναμένη*; and that in this is to be found the true explanation of the obscurity of the passage." *Greek Geometry from Thales to Euclid*. Allman, p. 208.

"Commensurable in square is in the Greek *δυνάμει σύμμετρος*. In earlier translations (e.g., Williamson's) *δυνάμει* has been translated 'in power' but as the particular *power* represented by *δύναμις* in Greek geometry is *square* I have thought it best to use the latter word throughout. It will be observed that Euclid's expression *commensurable in square only* (used in Def. 3 and constantly) corresponds to what Plato makes Theætetus call a square root (*δύναμις*) in the sense of a surd." *The Thirteen Books of Euclid's Elements*. T. L. Heath. Vol. III, p. 11, Def. 2.

NOTE II.

THE large pattern of pavement blocks in the floor of the orchestra of the Theatre of Dionysos measures 13.225 by 7.635 meters. The lesser divided into the greater gives the ratio root three, 1.732. The joints of the stones which constitute this pattern are now more open than they were originally but these separations are uniform, so the ratio is not affected.

In the center of this large area there is another smaller area of similar shape measuring 2.115 by 1.221 meters. In the center of this smaller figure there is a rectangular block of stone with a circular depression at its center. This block measures .695 by 1.0425. The proportion of this stone is one by one and a half, that is, it is static. The diameter of the hole is one-half the length of the stone in which it is cut; again static.

Each side of the large diamond-shaped figure is divided into thirty-one equal parts.

It is clear, from the measurements and the drawing, that this pattern and all its subdivisions, with the exception of the rectangular stone at the center, are composed of



Figure 96.

Theatre of Dionysos. Pattern in the floor of the orchestra.

equilateral triangles as ACB or BCD of the diagram. The entire scheme is a prototype of the equilateral triangle patterns of the art of the middle ages and is distinctly different from the symmetry of classic Greek design.

The Greeks used a root-three rectangle but not in the manner of the Roman pavement pattern. The diagonals of a root-three rectangle make two equilateral triangles but they are in reverse position as in the diagram.

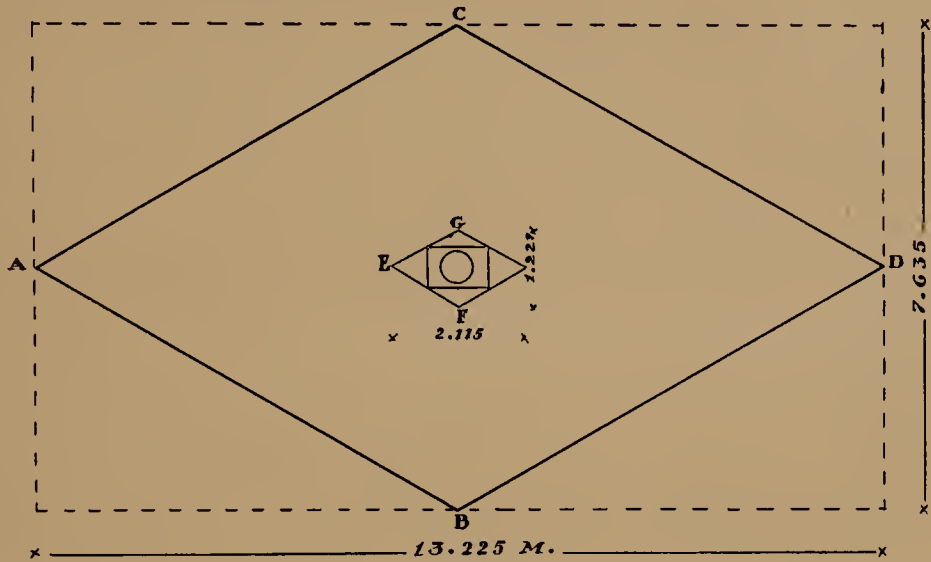


Figure 95.

Plan of the pavement pattern in the floor of the Theatre of Dionysos at Athens.

AB is a root-three rectangle. ADC and CBE are two equilateral triangles formed by the diagonals of the rectangle. The early Greeks used the proportions of root-three rectangles but not the patterns made by equilateral triangles. These figures would have been too obvious, too commonplace.

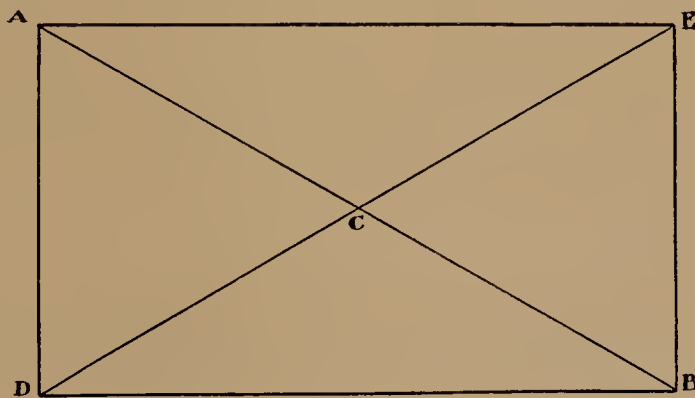


Figure 97.

A root-three rectangle and the Greek method of division.

NOTE III.

THE earliest potters whose work has survived in sufficient examples to supply knowledge of the type of symmetry which they used in their designs are:

AMASIS, a metic with an Egyptian name, who is supposed to have come to Athens from Ionia and to have worked there about 540 B.C. There are two examples of this master's designs in the Museum of Fine Arts, Boston, one in the Louvre, and one in the Bibliothèque Nationale in Paris. All of them are carefully worked and show simple dynamic schemes of the highest type. (See *The Diagonal*, July-September, 1921.)

TLESON, whose early work seems to have been static, as he used simple squares to fix proportions. His later work is dynamic. At least the static designs are not so good as the dynamic creations and, as there is a transitional example which exhibits both types, it seems fair to assume that the static examples are earlier. There are examples in the Museum of Fine Arts, Boston, British Museum, the Museum at Cambridge, England, and the Louvre. He belongs to the group of designers known as the "Little Masters."

EXEKIAS, apparently connected with the same school as Tleson. Like the latter he seems to have confined his proportion schemes to root two. Fine example in the Louvre.

NIKOSTHENES. This master must have been a prolific worker, judging by the number of his designs which have survived—over sixty in all. There are twenty-five specimens in the Louvre alone. Fortunately they are exhibited in a group so a comparison of their design qualities may be made by inspection. Fifteen of these are unmistakably static. There are certain peculiarities about the composing elements of these designs which would seem to preclude a mistake being made in the classification. Ten examples of the group are as unmistakably dynamic, most of these being of the highest type. All of the Louvre specimens were measured by a French expert (who was unfamiliar with the dynamic theory) while I was away from Paris, so there was hardly an opportunity for any errors being made in favor of the theory. (For measurements see *The Diagonal*, July-September, 1921.)

In the museum at Florence, Italy, I found a beautiful Nikosthenes kylix of dynamic proportions.

In the Villa Giulia collection at Rome there are three specimens of this master's work. Two of them are static and of the same design type as similarly proportioned vases in the Louvre. The third example is a wonderful pyxis. It is one of the most striking specimens of dynamic symmetry yet found. In the Metropolitan Museum, New York City, there is a large Nikosthenes kylix which suggests a transition from the static to the dynamic type of symmetry.

ANDOKIDES, who flourished probably toward the end of Nikosthenes' career, seems to have been careful as well as ambitious. Examples of his fabrics are in the Louvre and are extraordinarily bold and beautiful. He marks what is known as the transitional period of pottery decoration as some of his vases are embellished in both the black- and the red-figured styles.

The work of the potters seems to show that dynamic symmetry was used in this craft about 540 B.C., and from that date there appears to have been scarcely a break in the usage until shortly after the great plague at Athens. During the early part of the fourth century B.C. there are decided lapses. By Roman times the classic thread is lost and pottery design is heavy, clumsy, and stupid.

NOTE IV.

DIFFERENCES between front and flank column separations are characteristic of early temples in Greece proper. This type of variation has never been explained. The modern architect almost invariably makes these axial separations equal.

This equality becomes characteristic in Greek temple design during the Hellenistic period and it may mark the difference between static and dynamic design. A dynamic rectangle cannot always be subdivided into equal parts on ends and sides. Such equal divisions would define a mesh of squares and therefore, to some extent, be static.

The standard was probably a wider separation of the front column axes than those of the flanks though the reverse is often found in the colonies. For example: Temple "D" at Selinus, Sicily, has 4.37 for the fronts and 4.51 for the sides; Temple "F" 4.48 and 4.60; Temple "G" 6.52 and 6.62. The Basilica at Pæstum, Italy, 2.87 and 3.10, and the Poseidon temple 4.48 and 4.50. The Olympium at Girgenti, Sicily, 8.08 and 8.12.

The temple at Assos, in Asia Minor, has the front column separations greater than those of the sides but I am unable to find a dynamic plan in this building. It was a small structure and certain features of it are radically different from other Doric temples—sculptural decoration on the architrave with extreme archaic figures, for example. Also it appears to have been a two-stepped temple.

NOTE V.

THE spacing of the five regularly separated columns of the façades, from center to center, is as follows:

<i>East end:</i>	<i>West end:</i>
14.078	14.102
14.106	14.106
14.113	14.096
14.084	14.092
14.115	14.113
<i>Total</i> 70.496	<i>Total</i> 70.509

Mean separation 14.1005.

These measurements are the actual ones obtained by Penrose.

Theoretical separation 14.11±.

"A figure which would correctly explain the inclination of all the columns of the Parthenon is that of the framed rafters of a very steep roof, hipped at both ends, constructed on an oblong plan similar to that of the temple. The *hip rafters* would represent the axes of the angle columns produced, receiving at different heights the intersections of the three adjacent pairs of columns on each side of the angle. The ridge which would receive the intersections of the nine central pairs on each flank, would, on a scale equal to the real building, be 5856 feet in height above the pavement. With the exceptions of those at the angles the axes of the columns on each front or flank are parallel to one another; while the angle columns, participating in the inclinations of the two contiguous colonnades, have a greater inclination than the ordinary columns in the proportion of the diagonal to the sides of a square." *Principles of Athenian Architecture*. Francis Cranmer Penrose. Chap. IV (n.).

NOTE VI.

TEN measurements of the north step tread of the cella supply a mean of .3092 meters.

Twenty-nine measurements of the south step tread furnish a mean of .3252.

The measurements of the north step tread, beginning at the northwest corner and moving east, are:

Northwest corner:

.3180
 .3170
 .3185
 .3160
 .3150
 .3150
 .3140
 .2900 }
 .2900 }
 .2985 }

Total 3.0920, mean .3092

Southeast corner:

.3320
 .3300
 .3310
 .3280
 .3185
 .3180
 .3170
 .3230
 .3220
 .3215
 .3145
 .3275
 .3285
 .3285
 .3280
 .3250
 .3250
 .3255
 .3260
 .3270
 .3275
 .3260
 .3240
 .3250
 .3250
 .3280
 .3260
 .3260
 .3270

Total 9.4310, mean .3252

Difference between the means of the north and south steps .0160.

The bracketed measurements are opposite that part of the cella interior where the force of the explosion of Turkish gunpowder was greatest.*

These measurements were made by Dr. L. D. Caskey of the Museum of Fine Arts, Boston, November 15, 1920, and I wrote them down as he read them. An effort was made to take them approximately equal distances apart.

* Measurements inside the cella show a corresponding disturbance. At the west end, close to the west wall, the width is 19.052 meters. Closer to the center it is 19.057. Opposite to the point indicated by the step disturbances it is 19.065 and nearer the east wall 19.057.

The earliest Greek foot, the Æginetan, is supposed to be .3270 or .3280 meters in length. The preference seems to be for .3270. The Athenian foot adopted at the time of the great reform of Solon is supposed to be equal to .2962 meters according to Professor Hill. He says this is the width of the tread of the north step while .3280 is that of the south cella step. The actual measurements show but a rough approximation to this. (See *American Journal of Archæology*, 1912, XVI, pp. 535-558.)

Professor Hill says that the blocks "thus proved to have been put to an earlier use" (in the Parthenon destroyed by the Persians?) "are on the average 1.77 meters long (six Solonian feet) . . . it is the average length of twenty of the twenty-nine blocks of the lower step of the north side of the cella and of a number on the other sides. All these had doubtless been used in the older Parthenon." This would make the Solonian foot .295.

Doerpfeld, the German authority, says the width of the nave is 9.82 meters or thirty Æginetan feet while the column height equals thirty-two of these early units. The column height of the Zeus temple at Olympia is equal to that of the Parthenon column. If 9.82 equals thirty of these early feet the column height should be 10.475 meters or 34.368 feet. The greatest height which can be given to a Parthenon column is 34.253 for three angle columns and 34.244 feet for one. The dynamic proportion trellis calls for 34.245 plus. Penrose made the southeast angle column height 34.244. The middle columns of the façades he fixed at 34.218, the difference between these heights and those for the angle columns being due to the curvature of the stylobate and entablature. According to actual measurements at Olympia it seems impossible to allow more than 10.43 meters for the Zeus column height or 34.22 feet. The dynamic trellis gives a more rational explanation of these discrepancies than does any hypothesis in regard to ancient measures as the simple geometrical construction is made without reference to any unit of measurement.

However, if it is granted that thirty and thirty-two Æginetan feet do appear in nave width and column height of the Parthenon, rational lengths in these terms can hardly be applied to any other measurements in the building unless infinitesimal fractions are employed, such fractions being $\frac{7}{24}$, $1\frac{1}{32}$, etc., etc. In fact any dimension may be expressed in such terms within an error of about $\frac{1}{16}$ of an inch. Penrose actually had recourse to $\frac{89}{90}$ of a length. Length measurements of this type shed no light whatever upon design practice except possibly to show that the designer did not know what he was doing. We know that the Greeks were masters of proportion and that they regarded two lengths as ends and sides of rectangles. They also had polygonal numbers for all sorts of regular shapes. It is curious that so many names for numbers were in architectural terms. There were plinth numbers, beam numbers, and stele numbers. (See Heath's *Euclid*, VII, Def. 17, note.)

NOTE VII.

THE first Greek geometer of whom we have knowledge was Thales, a merchant of Miletus (640-546 B.C.). He is supposed to have visited Egypt and there acquired a knowledge of geometry and astronomy. Thales, however, is credited with being the creator of abstract geometry as no proof has been found that the Egyptians possessed more than a practical knowledge of the subject, merely enough to aid them in surveying, calculating the contents of granaries, or planning buildings.

The discovery of the celebrated theorem that an angle in a semicircle is a right angle

is attributed to Thales by Pamphila, a female historian who lived at the time of Nero and is supposed to have been either an Epidaurian or an Egyptian. This theorem represents the first discovery of a natural law, the first notice of constancy in a world of apparent change.

It is possible that this law of the semicircle was hit upon by contemplation of design as we are reasonably certain was the case in regard to the discovery of the relation between the squares on a side and a diagonal of a square.

Draughtsmen, designers, or plan makers must have known, from the earliest times, how to draw a rectangle and its diagonals. Slight consideration would have made it clear that these diagonals were bisected at their point of intersection, thus making it possible to describe a circle whose circumference would touch the corners of the rectangle. The diagonals would thus become diameters of the circle and half the rectangle; that is, a right-angled triangle would be inscribed in a semicircle.

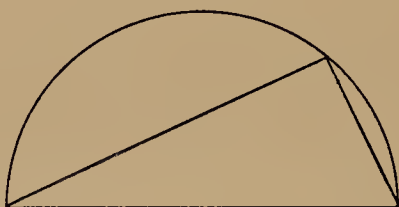


Figure 98.

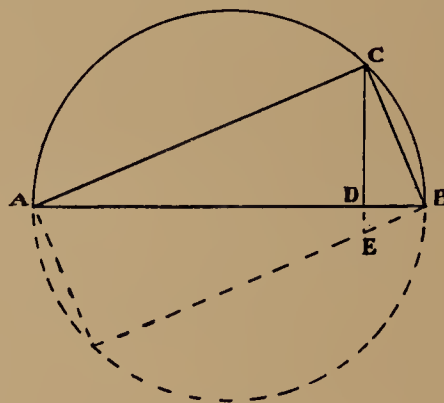


Figure 99.

Such a simple discovery would naturally suggest that no angle could be drawn in a semicircle except a right angle, as no four-sided figure except a rectangle could be inscribed in a circle with its center coinciding with the circle's center.

Another early and important discovery was the finding of a mean proportional line. This is CD of figure 99, a line drawn at right angles to AB to meet AC, BC at C. It seems to me natural that a plan maker would be inclined to consider the right-angled triangle ACB as half of a rectangle. In fact we know this was actually the case in India. The Hindus considered AB as a diagonal to a rectangle and not as an hypotenuse. This is made clear in the earliest record of their geometrical thought, the *Sulvasutra* or "Laws of the Cord." These laws were based upon the root rectangles and represented temple building ritual. I am assuming that this record is connected with temple building though it specifically refers to altar building. The two can hardly be separated.*

It seems reasonable to me that a geometer with a diagram before him, like that of figure 99, and, understanding that CD was a mean proportional between AD, DB, he might produce CD to E. If he did this and was a thinker of some caliber he might notice that, instead of three proportional lines DB, DC, DA, he now had four, DE, DB, DC, DA, and, instead of one mean proportional line, DC, he would have two, DB, DC.

* *Indian Mathematics*, by George Rusby Kaye, Calcutta and Simla.

One of the most celebrated geometrical problems of antiquity, as I have mentioned before, was that of the "Duplication of the Cube." This problem was ultimately reduced to one of finding two mean proportionals between two given lines. Allman has suggested that this problem may have arisen from the practical needs of architecture and afterward became a matter for scientific speculation. As he speaks of the problem having arisen in Theocratic times I take this to suggest that the priests may have used two mean proportionals in some of their architectural plans; that the philosophers, seeing what was so easily accomplished by the geometrical figures of the plans, were curious to know how two mean proportional lines could be found if the extremes only were given. That is, if DE, DA of the diagram were given, how could DB, DC be found? * Thus one of the most celebrated problems of antiquity could have arisen logically and naturally.

Some of the problems considered by the early explorers of science may seem strange and fanciful to us but we must remember that many of the subjects of these early speculations have proven to be vitally connected with the greatest of human achievements, and we are compelled to admit that there may be more in some of these ancient vagaries than appears on the surface. In connection with this notion of two mean proportional lines, for example, it may prove suggestive, and interesting, to the artist to quote an authority at some length.

"From the consideration of a passage in Plutarch . . . I am led to believe that the new problem—to find two mean proportionals between two given lines . . . had a deeper significance [than that of duplicating a cube], and must have been regarded by the Pythagorean philosophers . . . as one of great importance on account of its relation to their cosmology.

". . . the Pythagoreans believed that the tetrahedron, octahedron, icosahedron and cube corresponded to the four elements of the real world. This doctrine is ascribed by Plutarch to Pythagoras himself: Philolaus, who lived at this time held that the elementary nature of bodies depended on their form. The tetrahedron was assigned to fire, the octahedron to air, the icosahedron to water and the cube to earth; that is to say, it was held that the smallest constituent parts of these substances had each the form assigned to it. This being so, what took place, according to this theory, when, under the action of heat, snow and ice melted or water became vapor? In the former case the elements which had been cubical took the icosahedral form, and in the latter the icosahedral elements became octahedral. Hence would naturally arise the following geometrical problems:—

"Construct an icosahedron which shall be equal to a given cube;

"Construct an octahedron which shall be equal to a given icosahedron.

"Now Plutarch . . . accepts this theory of Pythagoras and Philolaus and in connection with it points out the importance of the problem: 'Given two figures, to construct a third which shall be equal to one of the two and similar to the other'—which he characterizes as essentially geometrical and attributes to Pythagoras. . . . It is evident that Plutarch had in view solid and not plane figures; for, having previously referred to the forms of the constituent elements of bodies, viz., air, earth, fire and water, as being those of the regular solids, omitting the dodecahedron, he goes on as follows:

" 'What' said Diogenianus 'has this (the problem—given two figures, to describe a third equal to one and similar to the other) to do with the subject?'

" 'You will easily know' I said 'if you will call to mind the division in the *Timæus*, which divided into three the things first existing from which the Universe had its birth;

* The line CE of figure 99 is a diagonal of a reciprocal to the rectangle AB.

the first of which three we call God ($\Theta\epsilon\acute{o}s$, the arranger) a name most justly deserved; the second we call matter, and the third *ideal form*. . . . God was minded then to leave nothing, so far as it could be accomplished, undefined by limits, if it was capable of being defined by limits; but (rather) to adorn nature with proportion, measurement and number: making some one thing (that is, the universe) out of the material taken all together; something that would be like the *ideal form*, and as big as the *matter*. So, having given himself this problem, when the two were there, he made and makes and forever maintains, a third, viz., the universe, which is equal to the matter and like the model.' " Allman, pp. 85-87. In this connection the reader should note that the five regular solids of geometry are dynamic in their proportions as is clearly proven in the Thirteenth Book of Euclid's *Elements*. See also Chapter III of *Dynamic Symmetry: The Greek Vase*.

This is the only definite reference to the nature of the Greek ideal which has survived, at least the only one I know. It is generally taken for granted that the Greeks were artistic idealists. As a matter of fact they were not, at least in the modern sense. The artistic idealists of today substitute themselves for nature. The classic Greek eliminated himself as much as possible and endeavored to let nature, at its best, tell the story. Actually he was a surrealist. Instead of brooding about himself he tried to understand the deeper meaning of natural phenomena and to become an unprejudiced interpreter. This is the message that Greek art has for me. The master is the best slave. The way to freedom lies through obedience to law. The Persian king marvelled at the bravery, lack of fear, and fighting ability of the Greek soldier without a master and the Spartan explained that he obeyed but one master, that was law.

NOTE VIII.

IN all the arts of form the law of symmetry takes precedence. That is, a design in two dimensions or in three, to satisfy the eye, must show due proportion of parts in relation to one another; grace, expression, the essential value of the work, reside largely in the manner in which space is filled and spaces are related to one another by the parts of the design. The space-relations may be very simple, as in a geometrical pattern like a rosette or a meander, or may be very complex and subtle, as in a fresco by Michael Angelo or a Phidian pediment group. In like manner the dance, poetry, music, are subject first of all to the law of rhythm, which is in the broadest sense due proportion of parts in time. Evidently rhythm and symmetry are but two aspects of one law, that of proportionality, acting in time or space according to the material concerned." *Athenian Tragedy*, by Thomas Dwight Goodell, p. 25. Yale University Press, 1920.

PROOF BY ARITHMETIC OF THE DIAGRAMS IN THE TEXT.

CHAPTER I.

Figure 4. Center to center of the angle columns of the flank 221.439 feet.

Intercolumniation of façades $14.106 \pm$ feet. Actual measurement 14.098.

The lesser of these measurements divided into the greater equals 1.42705×11 .

1.42705 equals .618 plus .809. The fraction .809 is one-half of 1.618, hence the arrangement of .618 rectangles of the diagram. (Note that .42705 is one-quarter of the façade rectangle.)



Figure 100.

The interior of the cella as seen from above. The black line roughly indicates the rectangle of the column centers. The darker poros stone base, with a hole in its center, shows the placing of the Phidian statue of Athena.

Figure 5. Center to center of the angle columns of the fronts 94.67 feet.

Flank intercolumniation 14.085 feet.

The lesser into the greater equals 6.72136 (2.36×2 plus 2).

Figure 6. Height from top of column to top of building is 24.89 feet; length of lowest step 111.31. Resulting ratio 4.472 (2.236 or root 5×2).

Length of entablature cornice, including corona and mouldings, 104.645 feet.

Ratio of AG is $.618 \times 12$.

Ratio of IG is 1.618×6 .

Ratio for CA or BD is 7.472, 1.618×4 plus 1 . Or it may be considered as 3.236 plus 4.236 .

Figure 7. Width of building 111.31 and length 238.003 feet. Ratio 2.1382.

A root-five rectangle in this position has the value .4472.

A frequently occurring rectangle in this symmetry is composed of a square and a root-five area, ratio 1.4472.

The reciprocal of this ratio, found by division into unity, is .691.

1.4472 plus $.691$ equals 2.1382. Thus, the area of the full ground plan is composed of a square and root-five rectangle plus a reciprocal.

Figure 8. Full width of building 111.31 and full height 65.162 feet.

Ratio 1.7082, that is, a square plus three .236 rectangles ($.236 \times 3$ equals .7082).

Figure 9. These areas are best calculated by using the reciprocal of 1.7082, that is, .58541, which will be the value of the line DB if AD is unity.

We may use .618 as a multiplier to divide DB in extreme and mean ratio.

$.58541 \times .618$ equals .3618.

$.58541$ minus .3618 equals .2236.

The former is the value of EB, the latter of ED.

.2236 is the reciprocal of 4.472 (root five, 2.236×2).

.3618 is the reciprocal of 2.764 ($.691 \times 4$).

By direct measurement BE is 40.272 and AD 111.31 feet.

Result by division 2.764.

DE is 24.89, AD 111.31 feet, ratio 4.472.

Figure 10. The superimposed rectangles of façade and ground plan ought to be sufficiently clear from the explanation given.

The relation of the square and root-five rectangle of the façade to that of GH of the ground plan is 2.764 to 1.

Figure 11. The ratio for the rectangle is furnished by 1.7082×2 plus .236, that is, 3.6524.

The full height, 65.162, divided into the full length, 238.003, equals 3.6524.

The reciprocal of the façade divided into the ratio of the ground plan, .58541 into 2.1382, equals 3.6524, or 2.1382×1.7082 produces the same result.

CHAPTER II.

Figure 12. The rectangle is 65.162 by 111.31 feet.

Ratio 1.7082. Reciprocal .58541.

BH equals 40.272 feet.

CA equals 24.89 feet.

$\frac{111.31}{24.89}$ equals 4.472, equals 2.236×2 .

FA equals 14.111 feet.

$\frac{111.31}{14.111}$ equals 7.88854.

1.7082×2 equals 3.4164.

4.472 plus 3.4164 equals 7.88854.

Figure 13. AC equals .2236 equals 24.89 feet.

CF equals .0968 equals 10.779 feet.

.0968 is the reciprocal of 10.326.

Root five $\times 4$ (2.236×4) equals 8.944.

.691 $\times 2$ equals 1.382.

8.944 plus 1.382 equals 10.326.

Figure 14. BD equals 51.0512 feet. AD equals 111.31 feet.

Ratio 2.18034.

.618 $\times 2$ equals 1.236.

2.18034 minus 1.236 equals .944.

.236 $\times 4$ equals .944.

Figure 15. BE equals 51.0512 feet.

BH equals 6.025 feet.

HE equals 45.026 feet.

.618 $\times 4$ equals 2.472.

45.026 $\times 2.472$ equals 111.31, the temple width.

Figure 16. DH equals 34.247 feet.

JK equals 84.658 feet.

34.247×2.472 equals 84.658 feet, equals six intercolumniations.

Figure 17. FG equals 34.247 feet.

EF equals 94.67 feet.

34.247×2.764 equals 94.6587, etc.

2.764 equals .691 $\times 4$.

Figure 18. One column-spacing rectangle, 14.1 by 34.247, provides the ratio 2.427 (2 squares and one-quarter of the façade rectangle), or .809 $\times 3$.

.809 equals two .618 rectangles, one on top of the other, that is, it is one-half of 1.618.

Figure 19. Façade rectangle equals 1.7082.

One-quarter equals .42705.

.42705 is the reciprocal of 2.34164.

Root five $\times 3$ ($.4472 \times 3$) equals 1.34164; plus a square it equals 2.34164.

Figure 20. Height 6.025; width 111.31. Ratio 18.472.

.236 $\times 2$ equals .472.

CHAPTER III.

Figure 21. The distance from the top of the entablature to the top of the sima is 14.111 feet. Building width 111.31 feet. Ratio 7.88854.

Length of entablature, including corona and mouldings, 104.645 feet (Penrose made it 104.634).

Resulting rectangle, with full pediment height of 14.111 feet, 7.4164, or .618034 $\times 12$.

7.88854 minus 7.4164 equals .472 or .236 $\times 2$.



Figure 101.

The northeast corner of the Parthenon steps, showing curvature of the podium. The small step is the Euthynteria or first levelling course. The proportions of the building were probably fixed by this course, although an analysis need not be confined to this feature of the step scheme. Any of the other steps furnish rectangles which produce the same scheme of proportions.

Figure 22. Perpendicular height of the sima 1.43 feet.

BF equals 14.111. Minus 1.43 equals 12.681.

104.645 divided by 12.698 equals 8.2404, that is, $.412 \times 20$.

.41202 is the reciprocal of the column-spacing rectangle of 2.427.

Hence there are ten column-spacing rectangles in half of the pediment projection.

There is an error of about two-tenths of an inch in the height but it is doubtful whether closer results than this can be obtained because of the uncertainty created by the curvature and contraction refinements.

Figures 23 and 24 are explained in the text.

Figure 25. The area CD has the ratio value of 10.326 (10.779 into 111.31 feet).

The area AB has the ratio value of 1.618×6 or 9.7082. 9.7082 also equals four column-spacing rectangles of 2.42705 each.

9.7082 subtracted from 10.326 leaves .618 for the two areas AC and BD, or .309 each.

Figure 26. The epistyle length as fixed by direct measurement is 100.295 feet, divided as follows: 15.263, 13.617, 14.126, 14.210, 13.889, 13.963, 15.227, total 100.295.

This is for the east front; it is a minute quantity shorter on the west.

The height of the entablature without the cornice is 8.842 feet.

Three root-five rectangles ($.4472 \times 3$) equal 1.34164; plus ten squares it is 11.34164. Error .013 feet.

Figure 27. The height of the epistyle, without the fillet, is 4.057.

100.295 divided by 4.057 equals 24.72136 or $.618034 \times 40$.

The frieze height is 4.788. With the length the ratio is 20.9472.

The fraction .9472 equals .5 plus .4472 or two squares plus a root-five rectangle, and the shape stands in a reciprocal position to the metope rectangle.

Figure 28. AD equals 3.96 feet and DB 4.185 feet. Ratio 1.056.

Reciprocal .9472.

FB is 3.84 feet. EG 4.484 feet. FE is 2.766 feet.

$\frac{4.484}{2.771}$ equals 1.618.

CHAPTER V.

Figure 42. The reciprocal of a column-spacing rectangle is .41202.

EB equals .41202.

BF equals 1.42705.

FD equals .41202.

Ratio $\frac{1.42705}{.41202}$ equals 2.25109.

101.347×2.25109 equals 228.141, the top step length of the temple.

Figure 44. AD is the top step width, 101.347 feet.

AB is a .618 rectangle.

$101.347 \times .618$ equals 62.632.

GE equals .809 equals 81.989 feet.

81.989 plus 62.632 equals 144.621 feet.

AM, KB equal IH. JF equals AI equals MD.

KH equals 62.632 by 144.621.

Ratio of KH is 2.309.

AI equals 3.236, GF equals 4.236 (see pediment and entablature excess).

Figure 45. GH equals 15.88 and EF 15.878 feet.

KI equals 24.60, NJ 24.15 feet.

Figure 46. Measurements on diagram.

CHAPTER VI.

Figure 47. Width of temple floor 101.347 feet.

Height from floor to ceiling 43.28 feet.

Ratio 2.34164, reciprocal .42705.

(Root five $\times$ 3 equals 1.34164.)

Rectangle CD 43.28 by 73.36 feet.

Ratio 1.6944 equals .4236 $\times$ 4.

.4236 is the reciprocal of 2.36068.

Figure 48. Reciprocal of column-spacing rectangle .41202.

Multiplied by 4 this is 1.64808.

43.28 $\times$ 1.64808 equals 71.33 feet.

Figure 49. Height of cella steps 2.291 feet.

Height of FG 41.00 $\pm$ feet.

.618 $\times$ 4 equals 2.472. Error .010.

$\frac{101.347}{2.472 \pm}$ equals 41.00 feet.

Width of top step 73.36 $\pm$ feet.

Root five $\times$ 4 equals 1.788854 (.4472 $\times$ 4).

41.00 $\pm$ $\times$ 1.788854 equals 73.36 $\pm$.

Figure 50. Length of cella upper step (flank) 193.733 feet.

Height from temple floor to ceiling on flanks 43.28 feet. Error .02.

Resulting rectangle 4.472 (root five $\times$ 2).

Figure 51. Length of cella upper step 193.733 feet.

From cella floor to ceiling on flank 41.00 feet.

Resulting rectangle 4.72136 equals 2.36068 $\times$ 2.

Figure 52. Length of upper step 193.733 feet.

Height of columns for flank 33.093 feet.

Resulting rectangle 5.8541 equals .58541 $\times$ 10.

.58541 is the reciprocal of 1.7082, the temple façade rectangle.

Figure 53. Width of upper step 71.36 feet.

Column height, front, 33.093 feet.

Rectangle 2.1572. $\frac{1.618}{3}$ equals .5393.

.5393 $\times$ 4 equals 2.1572. Error .02 in width of step.

.5393 is the reciprocal of 1.8541 or .618 $\times$ 3.

Figure 54. Length of the bottom step (flank) 196.383 feet.

Cella floor to ceiling (flank) 41.00.

Ratio 4.788854. Error in length .04.

Width of bottom step (fronts) 73.36 feet.

Ceiling height on fronts 41.00 feet.

Ratio 1.788854. Error in height .009 feet.

The proportion trellis shows a slight variation between the front and flank elevation



Figure 102.

Steps at the west end of the Parthenon which were cut in the rock of the Acropolis. These steps are curved in elevation and furnish proof that the system of curves found in the temple was intentional.



Figure 103.

Present condition of the southeast corner of the cella showing the foundation block (of marble), the upper surface of which was cut to receive the orthostate block. The setting line, fixed by the builders, is also clearly shown on adjoining foundation blocks.

heights which is either due to variations of curvature between the floors and ceiling or to the uncertain lengths and widths of the steps.

These minute variations may, also, be due to a difficulty which the masons encountered when they tried to adjust their work to the varying curvature of the fronts and flanks. Such a difficulty would be overcome practically by trimming the stones *in situ*.

The first masonry course of the walls is considerably shorter in the middle than it is at the ends so that the courses of stones at eye level are practically straight. This probably was an attempt to simplify the stone cutting for the walls.

The curvature may have been taken up again near the ceiling but evidence for this, other than that supplied by the proportion trellis, is lacking.

At any rate the floor of the Parthenon is like the curve of an enormous lens, of low magnifying power, cut like a rectangle. This would produce automatically a greater curvature on the flanks of the cella than on the fronts.

Drawing showing members between the floor and the ceiling with Penrose's measurements.

CHAPTER VIII.

Figure 58. The ratio for a square and four root-five rectangles is 1.559017. The fraction is equal to one-fourth of root five (2.236068).

The width from wall face to wall face is 62.953 feet.

The length from wall face to wall face is 98.145 feet.

The lesser into the greater measurement is 1.559017.

Figure 59. Penrose found that the mean column separation, from center to center, was 8.55 feet. There are ten strips across the major rectangle, counting the east and west strips as equalling two, so that the total length is 85.5 (by calculation 85.5012).

The width, 36.22, is a trifle narrower than Penrose made it, but 36.22 is what Doerpfeld found and I verified this by many measurements. This distance has received much attention as Doerpfeld uses it to back up a theory he had about the Greek foot.

Figure 60. Modern investigators center the columns exactly where the foundation blocks join. These blocks are not quite the same length so that their junctures can hardly mark the true centers. The center of the second column from the east wall on the north flank is clearly fixed by a circular roughening of the foundation blocks. The juncture of the blocks is some distance away from the center of the circle.

The width of the poros stone foundation for the statue platform varies slightly as some of the edging stones are wider than others. The builders evidently preferred to cut the soft poros stone blocks to fit rather than the harder marble blocks. The photograph of the edge of the poros stone rectangle shows this clearly, also that the rectangle width equals one intercolumniation.

From the centers of the west angle columns to the centers of the next columns to the east the distance is 9.56 minus, feet.

From the east wall to the centers of the next columns to the west the distance is 7.54 minus, feet.

Regular mean intercolumniation 8.55012 feet.

8.55012×8 equals 68.40096.

9.56 plus 7.54 equals 17.10024, which equals 8.55012×2 . Total 85.5012.

Figure 61. Distance from the center of the statue to the column center line AF is 22.385 feet.

36.22, length of AF, $\times .618$ equals 22.384. Error .001.

8.55012 divided into 36.22 equals 4.236, of which .236 is the reciprocal.

4.236 equals root five plus two squares.

JH equals two squares. KB plus EJ equals a root-five rectangle.

9.56 minus, distance of AB, divided into 36.22, distance of AF, equals 3.788854.

Root five $\times 4$ (.4472 $\times 4$) equals 1.788854 and plus two squares equals 3.788854.

Figure 62. 1. plus root five divided by four equals 1.559017.

One-third of root five is .745356.

1.559017 plus .745356 equals 2.304373, which is the ratio for the whole interior of the cella.

62.953 $\times$ 2.304373 equals 145.057 feet, the total length of the cella from east to west wall (fixed by the orthostate blocks).

Figure 63. The column-centering rectangle of the Maiden's room is 15.88 by 22.66 feet; ratio 1.427, which equals .618 plus .809.

Penrose made the width of the cella interior 63.01 feet. This is not quite accurate as both Doerpfeld's measurements and mine show. At the southeast corner of the interior there is a foundation block which was actually cut away to receive the first wall course block. This cutting, which is curious and appears nowhere else in the building, fixes the exact width of the plinth under the wall and establishes the true width of the cella interior. (See photograph.)

CHAPTER IX.

Figure 68. Penrose left no published measurements of the antefix. During 1902, a few years before his death, he gave me a tracing of a volute with a few measurements in feet and hundredths on it. I found afterwards that these agreed with the originals but there were not enough of them to justify the use of the English foot to record the data; therefore I will use the meter for the antefix as I did for the Panathenaic frieze.

Length of DB 0.38 $\pm$.

Length of AD 0.2626 $\pm$.

Ratio 1.4472, *i.e.*, 1. plus a root-five rectangle.

Height of spiral 0.19 $\pm$.

From eye center to eye center 0.19 $\pm$.

Total width 0.38 $\pm$. Ratio 2.

CB is 0.0726 $\pm$.

BD is 0.38 $\pm$. Ratio 5.236 (1.618 $\times$ 2 plus 2).

Figure 69. Height of rectangle 0.19 $\pm$.

Width of rectangle 0.3245 $\pm$.

Ratio 1.7082, that of the façade rectangle.

Figure 70. Distance from center to center of the eyes 0.19 $\pm$.

Other measurements and ratios are given above.

Figures 71 and 72. Simple constructions.

Figure 73. Total height 0.5175.

Total width 0.38. Ratio 1.3618.

The fraction .3618 represents an area similar to that part of the façade rectangle

made by the temple width and height from the ground to the top of the columns. AC is composed of a root-five rectangle and a half.

The ratio is .6708, *i.e.*, .4472 plus .2236.

The reciprocal of a square and a root-five rectangle is .691. This plus .6708 equals 1.3618.

FE is a similar figure to one-half of the façade area. The façade area is 1.7082. One-half is .8541.

.8541 is the reciprocal of 1.17082.

Reciprocal of 5.236 (EB) equals .191.

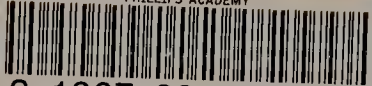
1.17082 plus .191 equals 1.3618. Compare with .8541 spiral.

Fragments of antefixes lying behind the Acropolis Museum have eye center separations nearly a centimeter shorter than those of the originals here given. The originals in Athens and in the British Museum probably occupied a position near the east or west ends of the eaves. The examples with narrower eye separation probably were fixed near the middle of the eaves. The fragments mentioned give smaller measurements for all the parts which exist, therefore I suspect the middle antefixes were similar figures to the others but smaller, made so to fit the curve of the flank cornice. The tops of the antefixes were probably on a straight line.



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